QF101 Quantitative Finance

Mid-Term Test

1. The objective of this mid-term test is to help you revise and absorb the key concepts discussed from Week 1 to Week 7, which are, respectively, in correspondence with Problems 1 to 7. Problem 8 is to help you reflect on the topics, which include quant's ways of thinking and problem solving, as seen in my weekly answers/feedback to the assignment exercises, additional exercises, additional examples, and weekly puzzles.

2. The take-home test problems are released on Oct 7 (Friday) 11 AM on elearn and on my web site. You are free to refer to the notes and the textbook to solve the problems.

3. You have about three days to prepare your answer scripts. Please submit your answer scripts to me personally on Oct 10 (Monday) from 10:30 AM to 5 PM. My office is Room 5036, LKCSB Level 5.

4. Late submission will not be accepted, since you are given ample time to work on the take-home test, and more importantly, the feedback/answers to the problems will be released on Oct 10 5 PM.

Problem 1. According to a Reuters' report, a record number of 10,149 hedge funds are in the HFR database as of end of March 2015.

(A) From 1988 through 2013 (see Slide 31 of Week 1 Introduction to Quantitative Finance), Medallion Fund's has only one down year. Assuming that hedge fund is equally likely to make or lose money, what is the probability of finding a hedge fund to achieve this feat?

(B) How many more hedge funds are needed in the database in order to match the probability of finding a hedge fund as stellar as the Medallion Fund?

Problem 2. The following table of FX quotes are taken from FOREX.com at the end of the week, October 1, 2016.

PAIR	BID	ASK
EUR/USD	1.12373	1.12437
GBP/JPY	131.435	131.612
GBP/USD	1.29687	1.29781
USD/JPY	101.310	101.391

You start from €1 million. Convert it to USD, and then convert USD to JPY, JPY to GBP, GBP to USD, and finally USD back to EUR. Compute the P&L of this round trip.

Problem 3. Suppose the world is binary. The stock price either rises to \$26 or drops to \$24 from the current price of \$25 ($= S_0$). The risk-free interest rate is 2% for tenor T, and the compounding scheme is continuous (exponential). Suppose a bank structures a financial contract whose payoff depends on the stock price S_1 at time T, which is a year:

$$\text{Payoff} = \begin{cases} \$100, & \text{if the stock price goes up to \$26 per share;} \\ \$10, & \text{if the stock price goes down to \$24 per share.} \end{cases}$$

(A) Determine the number of shares x and the amount of bond y needed to replicate the payoff.

(B) What is the present value of this derivative?

(C) What is the risk-neutral probability of $S_1 = \$24$?

Problem 4. Consider a parsimonious model of the term structure of interest rates:

$$Y_T = r + \beta^{(l)} T - \beta^{(s)} \left(\frac{1 - e^{-T/\tau}}{\frac{T}{\tau}} - e^{-T/\tau} \right).$$

(A) Show that the level of the yield curve is r.

(B) Show that the gradient of the parsimonious model by differentiation with respect to T is

$$Y_T' = \beta^{(l)} - \beta^{(s)} \left(e^{-T/\tau} \left[\frac{1}{T} + \frac{1}{\tau} \right] - \frac{\tau}{T^2} \left(1 - e^{-T/\tau} \right) \right).$$

(C) What is the gradient of the parsimonious model when $T \to 0$?

(D) What is the gradient of the parsimonious model when $T \to \infty$?

Problem 5. The (annual) zero rates z_i are 1%, 1.2%, 1.3%, and 1.4% for $i = 1, 2, 3, 4$. Each i is a half year, i.e. $i = 2$ is a year. Using Equation (6) in Slide 23 of Week 5 “NPV”, compute the discount factors.

(A) Price an interest rate swap that matures in one year. (Hint: Equation (11) in Slide 29 of Week 5)

(B) What is the par rate c that corresponds to the tenor of 2 years? (Hint: See Slide 34 of Week 5)

Problem 6. Take the data from Slide 35 of Week 7. Suppose the relevant (annual) risk-free rate is 0.5%. Note that $S_t = 70.61$ and $T - t = 24/365$.

(A) Compute the average of the bid and ask quotes for ITM calls and apply the put-call parity (Equation (1) in Slide 23) to obtain the prices for two OTM puts struck at 70 and 67.5.

(B) Compute the average of the bid and ask quotes for ITM puts and apply the put-call parity (Equation (1) in Slide 23) to obtain the prices for two OTM calls struck at 72.5 and 75.

Problem 7. The static replication of a payoff function $f(S)$ at contract maturity or expiration is

$$f(S_T) = f(\lambda) + f'(\lambda)(S_T - \lambda) + \int_0^{\lambda} f''(K)(K - S_T)^+ dK + \int_{\lambda}^{\infty} f''(K)(S_T - K)^+ dK.$$

(A) Apply the first principle of $\mathbb{Q}$F to show that at time 0, the present value of the payoff $f(S_T)$ is

$$\text{PV} = f(\lambda)e^{-r_0 T} + f'(\lambda)\big(c_0(\lambda) - p_0(\lambda)\big) + \int_0^{\lambda} f''(K)p_0(K)\, dK + \int_{\lambda}^{\infty} f''(K)c_0(K) dK.$$

(Hint: Think deeper about Equation (3) in Slide 14 of Week 3, Principle of $\mathbb{Q}$uantitative Finance.)

(B) Your client wants a payoff function of $f(S_T) = \left(\frac{S_T}{S_0}\right)^2$ in a year's time, i.e., T=1. The strike prices K of an option chain of maturity T is \$4.00, \$4.25, \$4.50, ..., \$7.50, \$7.75, \$8.00. The prices of puts and calls are, respectively, $p_0(K)$ and $c_0(K)$. What is the minimum price you will quote for your client for each contract of the required payoff when the risk-free rate is 1% and S_0 = \$6.00? (Hint: Let $\lambda = F_0$. Express your answers in terms of $c_0(K)$ and $p_0(K)$ for K = 4, 4.25, ..., 7.75, 8.)

Problem 8. You have the choice to answer either (A) or (B). The requirements are 100 (minimum) to 300 (maximum) words.

(A) If you are taking (or have taken) FNCE101 Finance, compare and contrast it with QF101 Quantitative Finance.

(B) Discuss what you find is the most interesting topic in QF101 Quantitative Finance.