

Mid-Term Revision of Quantitative Finance 101

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Underlying vs. Derivative

Underlying Asset

- ✂ No defined expiration
 - Fiat money
 - Equity
 - FX
 - Spot commodity

- ✂ Defined expiration
 - Fixed income

Derivative

- ✂ Linear payoff
 - Forward and Futures
 - Swap

- ✂ Nonlinear payoff
 - Plain Vanilla
European option
Structured warrants
 - American option
European option + early
exercise option

Mathematics Needed

Pre-U Math

- ✂ Algebra x
- ✂ Function $f(x)$
- ✂ Series $\sum_{i=0}^n a_i$
- ✂ Binomial theorem
- ✂ Limit $\lim_{x \rightarrow a} f(x) = f(a)$
- ✂ Differentiation
 - Chain rule
 - Product and quotient rules
- ✂ Maclaurin's series
- ✂ Integration
 - Change of variables
 - Integration by parts

U Math

- ✂ L'Hôpital's rule
- ✂ Taylor expansion
- ✂ Partial differentiation
- ✂ **Step function**
- ✂ **Dirac's delta function**
- ✂ **Riemannian integral**
- ✂ **Bivariate Taylor's expansion**
- ✂ **Bivariate integration**
- ✂ **Jensen's inequality**
- ✂ **Itô's calculus**

Three Principles of Quantitative Finance

First Principle

✂ 100% certainty

✂ Risk-free rate r

✂ $PV \xrightarrow{r} FV$

✂ $PV = e^{-r_0 T} \mathbb{E}^Q(\text{payoff function})$

Second Principle

✂ Uncertainty \rightarrow Risk

✂ Risky asset's $\mathbb{E}(r_r) > r$

✂ Risk premium

✂ $\text{Risk} \xrightarrow{r} \mathbb{E}(r_r)$

✂ $\mathbb{E}(r_a) \geq \mathbb{E}(r_b), \quad \text{if } \mathcal{R}_a \geq \mathcal{R}_b.$

Third Principle

✂ Buying = Selling

✂ “Law” of one price

✂ No risk-free arbitrage

✂ Market mechanism of fair price

Week 1

✂ Inspiration by a quantitative hedge fund: RenTech's Medallion Fund

- The wave of the future

✂ Revision of Pre-U Math through the Life of Pi

- Factorial $n!$
- Function $f(x)$
- $\sin(x), \cos(x)$
- n -th order derivative $f^{(n)}(x)$
- Integration
- Proof by contradiction

✂ Pre-U Math App 1: Binomial theorem, probability

✂ Pre-U Math App 2: Average

✂ Focus on grace rather than grade.

✂ Reward your efforts and processes.

Week 2

- ✂ Fundamental theorem of calculus
- ✂ Telescopic technique for sum (and product)
- ✂ P&L and return: idea of cash flow
- ✂ Arithmetic return vs. geometric return
- ✂ Models of stock prices
 - Deterministic exponential function
 - Geometric Brownian Motion
- ✂ Forex convention and market practice

Week 3

- ✂ Risk-free coupon bond
- ✂ Present value of “guaranteed” future cash flows
- ✂ Application of Pre-U Math’s geometric series to obtain a compact form of bond price as a function of yield y , coupon rate c , and number of years n
- ✂ Coupon bond selling at par, at discount, at a premium
- ✂ Capital vs. money market
- ✂ Defaultable bond
- ✂ Three principles of QF

Week 4

- ✂ Risk-neutral pricing
- ✂ Payoff replication
- ✂ Models following principle of $\mathbb{Q}F$
 - CAPM
 - Modigliani-Miller Proposition II
- ✂ **Relative valuation** from third principle of $\mathbb{Q}F$
- ✂ Model risk
- ✂ Derivation of CAPM by Pre-U Math's differentiation
- ✂ Continuous compounding
- ✂ Zero-coupon bond
- ✂ Long-term and short-term risk

Week 4

- ✂ Treasury yield curve
- ✂ Long-term risk
- ✂ Short-term risk
 - Monetary policy risk
 - Reinvestment risk
- ✂ Coupon bond = Portfolio of zero bonds
- ✂ Bond return = return from passage of time + return due to change in interest rate
- ✂ Duration and modified duration
- ✂ Convexity and modified convexity
- ✂ Level, slope, and curvature of yield curve
- ✂ Implied interest rate risk
- ✂ A Parsimonious model of yield curve

Week 6

- ✂ Self-financing strategy
- ✂ Derivation of fair forward price
- ✂ Derivation of fair forward FX contract
- ✂ Interest rate parity
- ✂ FX swap
- ✂ Non-deliverable forward
- ✂ Implied forward interest rate
- ✂ Forward rate agreement
- ✂ Fair FRA rate vs. LIBOR
- ✂ Discount factor
- ✂ Fixed and floating legs of IRS
- ✂ Long-short strategy of bonds

Week 7

- ✂ Overnight index swap as fixed rate
- ✂ Overnight (floating) rate as floating rate
 - Fed funds rate, EONIA, SONIA, CHOIS, TONAR
- ✂ CIRS
- ✂ Put and Call
- ✂ Intrinsic value vs. time value
- ✂ Moneyness: ITM, ATM, OTM, DOOM
- ✂ Plain vanilla payoff functions $(K - S_T)^+$ and $(S_T - K)^+$
- ✂ Obligation-right asymmetry \rightarrow option premium (price)
- ✂ Derivation of put-call parity $c_0 - p_0 = S_0 - e^{-r_0 T} K$
- ✂ Implied forward price K^* from $c_t(K^*) = p_t(K^*)$
- ✂ Modigliani-Miller proposition I
 - credit spread
 - irrelevance of capital structure

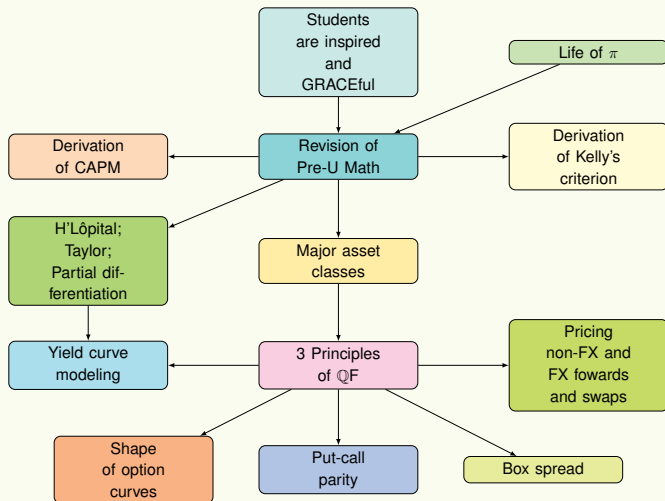
Real-World Examples

Singapore				Browse	13:52:43	Treasury & Money Markets			
SGD OIS		Swaps		3M FRAs		Spot FOREX		Deposits	
1M	0.350	1Y	1.145	0x3	0.875	SGD	1.3516	0/N	0.750
2M	0.400	2Y	1.347	3x6	0.975	EUR	1.1801	1W	0.938
3M	0.400	3Y	1.505	9x12	1.210	JPY	111.9100	1M	0.938
6M	0.450	4Y	1.645	6M FRAs		GBP	1.3281	3M	1.063
9M	0.450	5Y	1.763	0x6	1.045	AUD	0.7874	6M	1.100
1Y	0.500	7Y	1.985	3x9	1.135	Forwards		9M	1.200
		10Y	2.243	6x12	1.250	1W	-1.09	1Y	1.350
		15Y	2.457	SGD		1M	-4.77	Govt Bonds	
SGD/USD CCS		20Y	2.533	1M	1.00100	3M	-16.00	1Y	1.23 1.13
1Y	1.148	30Y	2.592	3M	1.12583	6M	-34.50	5Y	1.55 1.53
2Y	1.343			6M	1.25000	9M	-53.50	10Y	2.03 2.01
3Y	1.470			12M	1.37758	1Y	-74.00	15Y	2.32 2.29
6Y	0.000	Index Futures		Govt Bills		FOREX Cross		30Y	2.40 2.37
12Y	0.000	MSCI SING	370.85	3M	1.19	€/SGD	1.5951	Stock Indices	
SOR Fixing		STI	3325.37	1Y	1.24	JPY/SGD	1.2077	STI	3325.37
1M	0.840	Bond Futures		5Y	1.58	£/SGD	1.7950	HSI	28729.60
3M	0.925	1ST KB	0.00	10Y	2.07	AUD/SGD	1.0642	NKY	21287.22
6M	1.050	2ND KB	0.00	20Y	2.36	Overnight Rates			
						Average	0.69		
						Repo	0.21		

30 Economic Releases | ECO »

	Date Time	C	A	M	R	Event	Period	Surv(M)	Actual	Prior	Revised
31)	10/17 08:30	SI	.	.	.	Non-oil Domestic Exports SA MoM	Sep	-2.2%	--	4.5%	--
32)	10/17 08:30	SI	.	.	.	Non-oil Domestic Exports YoY	Sep	12.7%	--	17.0%	--
33)	10/17 08:30	SI	.	.	.	Electronic Exports YoY	Sep	15.0%	--	21.7%	--
34)	10/19 16:00	SI	.	.	.	Automobile COE Open Bid Cat A	Oct 19	--	--	41761	--

Flow of QF101 up to Week 9



Takeaways

- ✂ QF is based on 3 principles, applying math to re-look at finance with fresh insights and depth.
- ✂ Many important and useful models and formulas can be derived by applying the 3 principles of QF.
- ✂ QF101 deals with a wide range of financial instruments from cash (fiat money) to options.
- ✂ QF101 has many practical, industry examples (FX jargon, option chains, hedge fund, sovereign wealth fund, yield curves, OIS etc).
- ✂ The flow shows that QF101 is well organized and has a structured framework.