Answer. **Suggested answers for Q1**

(a) This question tests your understanding of the difference between bid and ask prices. To execute the triangular arbitrage, you as the trader will need to use S$1,710 to buy EUR at the ask price, which means that you will obtain €1,000, as €1,000 × 1.7100 = S$1,710, the amount you have.

Next, you sell €1,000 to obtain USD @ 1.3501 to obtain $1,350.10. Finally, you sell this amount of USD for SGD @ 1.2711, which will result in $1,350.10 × 1.2711 = S$1,716.11. Therefore, after one round of triangular arbitrage, the profit is S$1,716.11 - S$1,710.00 = S$6.11.

(b) It is clear that the bid price of USD/SGD, i.e., 1.2711, is too high. To find the appropriate bid, simply we solve for x such that you get back S$1,710 what you have started with:

$$1,350.10 \times x \leq 1,710.$$

We obtain $x \leq 1.26657288$. The idea is to find the largest x such that the inequality holds. Hence the bid price should be 1.2665 not 1.2666. One pip (0.0001) difference is enough to make the arbitrage works again!

□

Answer. **Suggested answers for Q2**

(a) The entropy $e(p) := -f(p) = -p\ln p - (1-p)\ln(1-p)$ is essentially the expected value (average) of log probabilities. To find the maximum, we take the first derivative of $e(x)$, i.e.,

$$e'(p) = -\ln p - 1 + \ln(1-p) + 1 = \ln(1-p) - \ln p.$$

The first order condition is that $e'(p) = 0$, that is

$$\ln p = \ln(1-p).$$

Applying exp on both sides, we obtain

$$p = 1 - p.$$

Hence $p = \frac{1}{2}$.

To prove that it is the maximum, we need to examine the second derivative, which is

$$e''(x) = -\frac{1}{1-p} - \frac{1}{p}.$$

Evaluated at $1/2$, we have

$$e''(1/2) = -4 < 0.$$

Accordingly, at $p = 1/2$, the entropy $e(1/2) = \ln 2$ is at its maximum.

(b) The answer is affirmative. In the binary system, the entropy is

$$e_2(p) = -p\log_2(p) - (1-p)\log_2(p).$$

Again, at $p = 1/2$, $e_2(1/2) = 1$, i.e., 1 bit. You need 1 bit to encode the information about the binary system at maximum entropy.

For intuitive reasoning, suppose only one of the two possible outcomes will always show up. In this case $p = 1$ and $e_2(1) = 0$; there is no need to encode the information because it is a sure event.

It is clear that $p = 1/2$ is the discrete uniform distribution for the binary system. As soon as p is either larger or smaller than $1/2$, $e_2(p)$ will be smaller, meaning that you can tell that one of the two events is more likely to occur, say 9 out of 10 ten times.

In the general n-nary system, again $p = 1/n$ has the highest possible entropy, for which every event is as likely to occur and thus highly uncertain.

Remark: In thermodynamics, entropy in a closed system is always increasing to the point where it reaches its maximum—every thing becomes uniform—when the equilibrium is reached. As an illustrative example, if you drop a tiny amount of ink into a glass of water, at first you see that the ink dye is located but after an hour (near certainty), you can't see anymore, because it is everywhere in the water (full uncertainty).

□

Answer. Suggested answers for Q3

(a) Instead of consuming one unit of goods and services (G&S) today, the rate at which the unit of G&S increases is the real rate h_0. You get compensated for withholding your consumption if h_0 is positive. In other words, instead of buying one unit at a price p_0 today, save (give up consuming) the money at the nominal rate of r_0 at a 100% trustworthy bank. In the next period, the cash allows you to buy $p_0(1+r_0)$ units. Use the cash to buy (and consume) G&S at π_1 per unit. Putting these ideas together, mathematically, when comparing the expected cash $p_0(1+r_0)$ at time 1 versus the expected price π_1, the ratio is related to the real rate as follows:

$$\frac{p_0(1+r_0)}{\pi_1} = 1 + h_0.$$

Moreover, we can write

$$\frac{p_0}{\pi_1} = \frac{1}{1 + \dfrac{\pi_1 - p_0}{p_0}}$$

The expected inflation is

$$i_0 = \frac{\pi_1 - p_0}{p_0}.$$

Incorporating these expressions into the ratio, we obtain

$$\frac{1}{1+i_0}(1+r_0) = 1 + h_0.$$

Consequently, Fisher's equation ensues.

$$1 + r_0 = (1 + h_0)(1 + i_0).$$

This equation allows you to infer the directly unobservable real rate h_0 from r_0 and i_0, which are observable.

(b) In the context of the textbook, it must be emphasized that the risk-free rate r_0 is free of default risk only. The Fisher equation indicates that the nominal r_0 is dependent on inflation rate i_0 and therefore subject to inflation risk.

The real risk-free rate should be h_0 from the purchasing power perspective. However, it is not directly observable. Nevertheless, Fisher's equation is consistent with the second principle of $\mathbb{Q}$F. To compensate investors for bearing the inflation risk, the investable nominal rate should be higher than the real risk-free rate. In this case, the risk premium is the inflation rate i_0.

Remark 1: Compared to nominal r_0, the real rate h_0 is much more important. That is why GIC, one of Singapore's sovereign wealth funds, has an investment objective of achieving a return higher than the inflation rate.

Remark 2: Economically, the real rate h_0 should be positive. Otherwise, inflation will erode the ability to purchase one unit of G&S. But short-term interest rates today are almost zero, implying that h_0 is negative as core inflation (food, education, health care etc) is still positive. The reality is inconsistent with the second principle of $\mathbb{Q}$F.

Negative real rate will tend to drive "rational" investors to invest more in risky assets, i.e., stocks, to fight inflation. When demand for equities is stronger, you will see that US stock market indexes (S&P 500, Dow Jones Industrial Average, and Nasdaq) will be higher. Low interest rates will also drive the fixed income prices higher.

That is why investors are wary of whether Federal Reserve is going to hike the target Federal fund rate or not this year (2016). Any hint from FOMC voting members who speak at various occasions will have a big impact on the market. For example, on Friday (September 9), **talk of a probable interest rate increase** by Boston Federal Reserve Bank President Eric Rosengren sent equity indexes tumbling by more than 2%. WTI oil and gold also sank. (see **Nightly Business Report - September 9, 2016**).

Remark 3: According to some economic model or reasoning, record low interest rate will lead to hyperinflation. But it is not happening (see Sections 2 and 3 of Chapter 1 in the textbook). In fact, Japan is in deflation for at least a decade. People don't behave like economic "agents" as the models suggest. People are people.

(c) Since the inflation and the real rates are small in magnitude, we can write the right side of the Fisher equation as

$$1 + h_0 + i_0 + h_0 i_0 \approx 1 + h_0 + i_0.$$

It follows that $r_0 \approx h_0 + i_0$. Hence

$$h_0 \approx r_0 - i_0.$$

This equation provides a quick way to compute the real rate h_0, which is the real risk-free rate of return after adjusting for inflation.

□