Answer. **Suggested answers for Q1**

(a) Refer to Page 27 in the textbook for bid and ask prices. In the ISO convention, to buy from the dealer 1 unit of base currency, which is GBP in this case, you as a customer need to pay more, i.e., \$1.5592 in this question. When you sell 1 unit of base currency, you will receive only \$1.5591. The difference in this example is \$1.5592−\$1.5591 = \$0.0001, which is called 1 pip in practice.

In general, let b_1 (e.g., \$1.5591) be the bid price and a_1 the ask price (e.g., \$1.5592). For selling (shorting) x units of GBP to the tune of \$1 million, you just need to solve the equation

$$xb_1 = \$1 \text{ million}.$$

Therefore $x = \dfrac{\$1 \text{ million}}{b_1} = \dfrac{\$1 \text{ million}}{1.5591} = £641,395.68$.

(b) To compute the P&L, you need to buy back x units of GBP. Since the quotes have become $b_2 = \$1.5580$ bid and $a_2 = \$1.5581$ offer, the cost is $xa_2 = £641,395.68 \times 1.5581 = \$999,358.61$. Therefore the P&L from trading GBP in dollars is

$$\text{P\&L} = \text{selling} - \text{buying} = \$1,000,000 - \$999,358.61 = \$641.39.$$

Remark 1: If you think of GBP as a stock, the calculation will make more sense.

Remark 2: The P&L should be positive because you sell GBP at a higher price of \$1.5591 and buy GBP at a lower price of \$1.5581.

Remark 3: For stocks, you as an investor usually buy first and then sell later; if you short a stock, you need to borrow shares from your broker. But in the FX market, you can easily (short-)sell first.

(c) Applying what you have read from Page 28 of the textbook, the return realized in British pounds is

$$\frac{\dfrac{1}{1.5581} - \dfrac{1}{1.5591}}{\dfrac{1}{1.5591}} = 0.0642\% = 6.42 \text{ bps}.$$

The acronym bps is "basis points". A basis point is 0.0001 or 0.01%.

(d) As in (b), the P&L in Greenback is \$641.39.

(e) The return in dollars must be the same. Indeed,

$$\frac{\$641.39}{\$999,358.61} = 6.42 \text{ bps}.$$

Note from (b) that \$999,358.61 is the buying price.

□

Answer. Suggested answers for Q2

(a) The simple return in dollars is

$$\frac{\$47.74 - \$38.13}{\$38.13} = 25.20\%.$$

(b) The simple return in Euros is

$$\frac{\frac{\$47.74}{1.2460} - \frac{\$38.13}{1.3591}}{\frac{\$38.13}{1.3591}} = 36.57\%.$$

The investor gains also from the appreciation of dollars. Buying a stock in a foreign market is also buying the foreign currency concomitantly.

□

Answer. Suggested answers for Q3

(a) A basic quality of a junior quant is the ability to formulate the problem mathematically. Suppose the asset price is P_0. Double that amount is $2P_0$. Given the rate of growth r, the forward value is $2P_0$.

$$P_0(1+r)^T = 2P_0$$

Taking logarithm on both sides, we obtain

$$T\ln(1+r) = \ln(2)$$

Now, applying your Pre-U math, $\ln(1+r) = r + O(r^2) \approx r$. Therefore,

$$T \approx \frac{\ln(2)}{r} \approx \frac{70\%}{r}.$$

(b) Given $r = 3.5\%$, primary school math leads to

$$T \approx \frac{70}{3.5} = 20.$$

The inflation is expressed as a percentage *per year*. Therefore, it was about 20 yeras ago.

□

Answer. Suggested answers for Q4

(a) First we write $\breve{r}_t$ as

$$\breve{r}_t = \frac{D_t}{P_{t-1}} + r_t.$$

Next, we consider

$$y_t \times \frac{P_t}{P_{t-1}} = \frac{D_t}{P_t} \times \frac{P_t}{P_{t-1}} = \frac{D_t}{P_{t-1}}.$$

Since $\frac{P_t}{P_{t-1}} = 1 + r_t$, we obtain

$$\breve{r}_t = y_t(1 + r_t) + r_t$$

Hence, we have shown that

$$y_t = \frac{\breve{r}_t - r_t}{1 + r_t}.$$

This formula is useful in computing the dividend yield from the time series of total returns $\breve{r}_t$ and simple returns r_t.

(b) Consider 2 dates: 1 and 2. Let date 2 to be the ex date and the prices are P_1 and P_2, respectively. Everything being equal, suppose there is no price change on ex date. In other words, $P_2 = P_1$. Then you buy at P_1 and sell it at the same price at P_2. By doing so, you pocket the dividend D_1. This is a simple trading strategy and everyone can do it easily. As a result, the price at date 1 is typically traded higher such that $P_1 = P_2 + D_1$. Therefore, $P_2 = P_1 - D_1$, in other words, on the ex date, the price must drop by D_1, everything else being equal.

□

Answer. Suggested answers for Q6

In *Temasek Review*, like many other annual reports, the definitions of 1-year, 5-year returns etc are not explicitly given. A basic quality of a junior quant is the ability to figure things out by making full use of the information available. Making an intelligent guess is often required.

(a) Guess: the 5-year return refers to the most recent five years. Hence, by the definition of geometric average, we solve for g as follows:

$$(1 + 1.5\%)(1 + 8.9\%)(1 + 1.5\%)(1 + 4.6\%)(1 + 0.42) = (1 + g_5)^5$$

The result is $g_5 = 0.1075 \approx 11\%$. Therefore, the claim is quite valid.

(b) Using the same method, the result is $g_{10} = 0.0859 \approx 9\%$. This claims is quite valid as well.

(c) The AUM for 2014 is S$223 billion. and for 2004, it is S$90 billion. Applying the simpler formula, the geometric average is

$$g_{10}^{*} = \left(\frac{223}{90}\right)^{\frac{1}{10}} - 1 = 9.50\%.$$

Likewise, the AUM for 2009 is S$130 billion. Accordingly,

$$g_{5}^{*} = \left(\frac{223}{130}\right)^{\frac{1}{5}} - 1 = 11.40\%.$$

(d) The difference of about 0.4% to 0.5% could be that in accounting for the total return, the costs of running Temasek Holdings and the estimated costs of divestments are factored in, resulting in a smaller value compared to the geometric average calculated from AUMs without considering costs.

Remark: This is just my guess. Only Temasek Holdings knows the reason.

(e) An important quality of a junior quant is the ability to perform reverse engineering. To formulate the problem mathematically, we let the unknown 1-year return be x, and write

$$(1 + x)(1 + 0.46) = (1 + 0.08)^2.$$

Solving for x, we find that $x = -20.11\%$.

Remark: Now you know why Temasek Holdings didn't release their annual report on 2003!

(f) Denote the 1-year return for 2002 by y. Accordingly,

$$(1 + y)(1 - 0.2011)(1 + 0.46) = (1 + 0.08)^3.$$

Solving for x, we find that $y = 8.00\%$.

(g) The standard deviation is computed from 8.0% for 2002, −20.11% for 2003, 46.0% for 2004, and so on. The number of observation is 13.

The unbiased standard deviation is computed and the result is 21.47%.

Given the assumption, the standard error is $\frac{21.47\%}{\sqrt{40}} = 3.3947\%$.

Therefore, the t statistic of the null hypothesis of 0% is

$$t = \frac{16\% - 0\%}{3.3947\%} = 4.71.$$

(h) A basic principle in statistic is not to discard away information. For the average 1-year total return of Temasek Holdings, we use all the 13 observations and we have $\overline{x} = 10.49\%$. For SIMSCI Index, $\overline{y} = 6.72\%$.

Earlier, we have computed the standard deviation for Temasek Holdings, which is $s_x = 21.47\%$. For SIMSCI Index, $s_y = 30.77\%$. With $n = 13$ and $m = 11$, the standard error (se) is computed as

$$\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}} = 11.02\%.$$

Consequently, the two-sample t statistic is

$$t = \frac{10.49\% - 6.72\%}{11.02\%} = 0.3421.$$

These two-sample t statistic shows that $\overline{x}$ and $\overline{y}$ are not statistically different.

Remark: Two-sample t test is a part of A level mathematics. Even if you have not done such test before, be aware that in practice, quants will encounter formulas that are not seen or taught. If the "manual" or documentation is sufficiently detailed, quants could easily follow and compute the required statistic. This question has the sufficient details for you to do.

□

Answer. **Suggested answers for Q9**

(a) From a purely investment's perspective, you have the fund to invest, and you decide to buy the bond, *having examined its yield*.

The buying price is 103% of the par value. At maturity, the issuer will pay you 100% of the par value, and the bond becomes value-less as it has just expired. In other words, you sell the bond at 100% of the par value and losing 3% in the process. Thus the simple return is

$$\frac{100\% - 103\%}{103\%} = -2.91\%.$$

But you receive 6% + 6% = 12% of the par value in the form of coupon during the two years when you are the bond holder. So the total return is

$$\frac{100\% + 12\% - 103\%}{103\%} = 8.74\%.$$

(b) This is a primary school question. The current yield is

$$\frac{6\%}{103\%} = 5.83\%.$$

Remark: What is the financial meaning of current yield? If you invest \$103 in a building and you collect rental of \$6. That is the yield of the building. Perhaps it is more insightful to think of a bond's current yield as stock dividend yield.

(c) The total return is

$$\frac{101\% + 3\% - 103\%}{103\%} = 0.97\%.$$

(d) Let x be $y/2$ where y is the annualized yield. We need to solve for x from the equation:

$$104 = \frac{3}{1+x} + \frac{3+100}{(1+x)^2}$$

By simple re-writing, it becomes

$$104(1+x)^2 - 3(1+x) - 103 = 0.$$

Moreover, let $z := 1 + x$. Consequently, a quadratic equation ensues.

$$104z^2 - 3z - 103 = 0.$$

By secondary school math, we have two solutions:

$$z_{\pm} = \frac{3 \pm \sqrt{9 + 42848}}{208}.$$

The solution z_- is negative and it does not make sense. So we consider only

$$z_+ = 1.009708$$

Therefore $x = z_+ - 1 = 0.009708$, and it follows that $y = 2 \times 0.009708 = 1.94\%$.

□

Think Like a Quant

Recall in Week 1, I talked about the importance of thinking like a professional quant. The reason is that in the Preface of the textbook, Page xv, I write from my experience in the industry that

> ... the gap between theory and practice is always wider in practice than in theory.

Some of you might find a gap in the materials discussed in class and the exercise questions. It is great that you have realized it! This is intentional on my part. Also, the exercises in the textbook are designed to be extensions of the main text. That's why you see some exercise questions are pretty lengthy and detailed.

The exercises are practice-oriented, in particular, the last two questions (Q6 and Q9). By contrast, other Finance textbooks have many many fictitious exercises and designed for drilling. If you have put your money at risk, you MUST know how to compute the P&L and returns correctly.

To make the exercises amendable, all the formulas needed for P&L calculation and returns are in the slides. In particular, geometrical average was discussed in class. It all boils down to knowning how to apply these formulas, which are, by all standards, very simple. Though simple, the skill of applying the formulas to solve problems correctly the very first time you do it takes time to develop.

My pedagogical approach is to let students get exposed to questions that they might not have encountered before. The objective is to train students to think and solve problems like a quant.

Most professional quants will tell you that they are trying to solve problems (Brexit, negative interest rates, global quantitative easing, etc) they have never encountered before. Textbooks are of little use because most's exercises are written for drilling, not so much for solving practical questions. Even if the problem is totally unseen before, quants must find a workable solution. They must do it right the first time they tackle it. In life, this is what happen in general—first time in marriage, first time being a parent, and the list goes on. So one of my goals is to let students learn how to bridge the gap, and to think like a quant.

The bottom line is that I want to see your efforts in solving the problems. You will awarded an A grade even if you cannot do some questions (but you must submit within the deadline). It takes time to develop the skill and to build the foundation, so hang on and perserve.