

Binomial Models

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Lesson Plan

- 1 Introduction
- 2 Random Walk
- 3 One-Period Option Pricing
- 4 Binomial Option Pricing
- 5 Nice Math
- 6 Takeaways

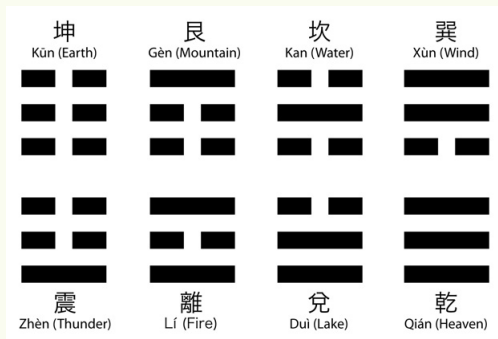
Applications of Randomness?

→ I Ching (Yi Jing)

→ Casting of lot



Source: [CARM](#)



Source: [brainpickings.org](#)

Binary Stochastic Processes

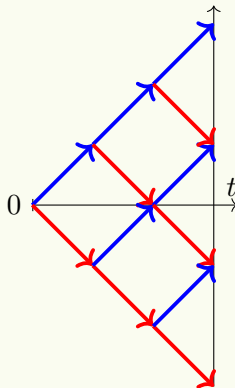
- Blank coin with blue dot on one side and red dot on the other side.
- Equally likely for either blue or red to turn up
- Random variable $C(\omega)$ is a mapping into numbers.

$$C(\omega) = \begin{cases} 1, & \text{if } \omega \text{ is blue;} \\ -1, & \text{if } \omega \text{ is red.} \end{cases}$$

3-Period Binomial Tree

$D_1 D_2 D_3$ $U_1 D_2 D_3$ $D_1 U_2 D_3$ $U_1 U_2 D_3$

$D_1 D_2 U_3$ $U_1 D_2 U_3$ $D_1 U_2 U_3$ $U_1 U_2 U_3$



One-Dimensional Random Walk

➡ A model of random walk S_t

$$S_t = S_{t-1} + C_t.$$

➡ Sum of up-down moves:

$$S_t = S_0 + \sum_{i=1}^t C_i$$

➡ Mean of C_t

$$\mathbb{E}(C_t) = \frac{1}{2} \times 1 + \frac{1}{2} \times (-1) = 0$$

➡ Variance of C_t

$$\mathbb{V}(C_t) = \frac{1}{2} \times (1 - 0)^2 + \frac{1}{2} \times (-1 - 0)^2 = 1$$

Mean-Variance Analysis of 1-D Random Walk

➡ Unconditional mean of random walk

$$\mathbb{E}(S_t) = \mathbb{E}\left(S_0 + \sum_{i=1}^t C_i\right) = S_0 + \sum_{i=1}^t \mathbb{E}(C_i) = S_0$$

➡ Unconditional variance of random walk

$$\mathbb{V}(S_t) = \mathbb{V}\left(S_0 + \sum_{i=1}^t C_i\right) = \mathbb{V}\left(\sum_{i=1}^t C_i\right)$$

➡ Since C_i and C_j are independent for $i \neq j$,

$$\mathbb{V}\left(\sum_{i=1}^t C_i\right) = \sum_{i=1}^t \mathbb{V}(C_i) = \sum_{i=1}^t 1 = t.$$

Conditional Mean of 1-D Random Walk

- ➡ At time t , S_{t-1} is known. Given S_{t-1} , what is the expected value of S_t ?

$$\begin{aligned}\mathbb{E}(S_t | S_{t-1}) &= \mathbb{E}(S_{t-1} + C_t | S_{t-1}) \\ &= \mathbb{E}(S_{t-1} | S_{t-1}) + \mathbb{E}(C_t | S_{t-1}) \\ &= S_{t-1}.\end{aligned}$$

- ➡ What is the interpretation of this conditional mean?

Answer: _____

Martingale

- ➡ Every coin toss C_i is **independent** of every other.
- ➡ Intuitively, the drunken man has no memory of where he has been before.
- ➡ Even if all information of the past is provided, still

$$\mathbb{E}(S_t | S_{t-1}, S_{t-2}, \dots, S_0) = S_{t-1}.$$

A fundamental theorem of financial mathematics

A financial market is viable (i.e., no risk-free arbitrage opportunity) if and only if there exists a probability measure under which the prices are martingales.

Conditional Variance of 1-D Random Walk

➡ Given S_{t-1} , what is the variance of S_t ?

$$\begin{aligned}\mathbb{V}(S_t | S_{t-1}) &= \mathbb{V}(S_{t-1} + C_t | S_{t-1}) \\ &= \mathbb{V}(S_{t-1} | S_{t-1}) + \mathbb{V}(C_t | S_{t-1}) \\ &= 0 + 1 \\ &= 1\end{aligned}$$

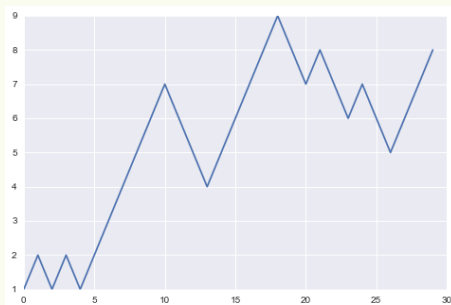
➡ What is the interpretation of this conditional variance?

Answer: _____

Where is the Random Walker After n Steps?

- ➡ Let n_+ be the number of upward moves, and n_- the number of downward moves.
- ➡ By definition, the total must add up: $n_+ + n_- = n$.
- ➡ Also,

$$S_n = S_0 + \sum_{i=1}^{n_+} U_i + \sum_{j=1}^{n_-} D_j = S_0 + n_+ - n_-.$$



Multiplicative Random Walk

Positive random variable A_t

$$A_t(\omega) = \begin{cases} u > 1, & \text{if } \omega \text{ is blue;} \\ d < 1, & \text{if } \omega \text{ is red.} \end{cases}$$

Multiplicative random walk

$$S_t = S_{t-1} A_t$$

Up to time t from time 0, the price of the asset is obtained as

$$S_t = S_0 \prod_{i=1}^t A_i. \quad (1)$$

Recombinant Binomial Tree

- ▶ The order of movements does not matter.

$$S_t = udS_{t-2} = duS_{t-2} = S_{t-2}.$$

- ▶ To make the binomial tree recombinant, you must have

$$u = \frac{1}{d}.$$

- ▶ Otherwise, for 100 periods, the number of end nodes S_T is $2^{100} \approx 10^{30}$.

- ▶ At each node, you compute $S_T - K$ for a call option. How long does it take on the fastest supercomputer? Given 33.86 Pflop/sec, which is 33.86×10^{15} , the time taken is

$$\frac{2^{100}}{33.86 \times 10^{15}} = 3.74 \times 10^{13} \text{ seconds} \approx 1,186,339 \text{ years!}$$

One-Period Case

At time 0, the underlying asset price is S_0 ,

At time 1,

$$S_1(\omega) = \begin{cases} uS_0, & \text{if } \omega \text{ is blue;} \\ dS_0, & \text{if } \omega \text{ is red.} \end{cases}$$

At time 0, construct a portfolio consisting a long position x_0 units of risky asset at S_0 per unit, and a short position in a call option at c_0 struck at K , for which $dS_0 < K < uS_0$.

The value (opposite of cash flow) of the portfolio is

$$V_0 = x_0 S_0 - c_0.$$

At time 1, the value of the portfolio is

■ If ω is blue,

$$V_1^+ = x_0 u S_0 - (u S_0 - K).$$

■ If ω is red,

$$V_1^- = x_0 d S_0.$$

Toy-Model Pricing

▶ To remove uncertainty and therefore risk, set $V_1^+ = V_1^-$

$$V_1^+ = x_0 u S_0 - (u S_0 - K) = x_0 d S_0 = V_1^-.$$

▶ Solving for x_0 ,

$$x_0 = \frac{u S_0 - K}{(u - d) S_0}. \quad (2)$$

▶ Therefore, the portfolio value at time 1 is also known at time 0, and we have

$$V_1 = V_1^+ = V_1^- = \frac{d(u S_0 - K)}{u - d} = \frac{S_0 - dK}{u - d}.$$

Toy-Model Pricing (cont'd)

- Since there is no uncertainty, by the first Principle of QF, the present value of the portfolio at time 0 must be discounted by the risk-free rate r_0 as follows:

$$V_0 = e^{-r_0} V_1 = \frac{e^{-r_0} d(uS_0 - K)}{u - d}.$$

We shall call e^{-r_0} the **risk-free discount factor**. But the portfolio value at time 0 is $V_0 = x_0 S_0 - c_0$

- Substituting in x_0 from (2), we find that the value c_0 of the call option should be

$$c_0 = x_0 S_0 - \frac{e^{-r_0} d(uS_0 - K)}{u - d} = \frac{1 - e^{-r_0} d}{u - d} (uS_0 - K). \quad (3)$$

Risk Neutrality

- ▶ In toy-model pricing, the probability p of upward movement by the underlying asset is not needed.
- ▶ By definition, the risk-free rate r_0 is indifferent to the up-or-down outcome. If the dollar value corresponding to the risky asset S_0 is invested in the risk-free bond, the value of this bond will become $e^{r_0}S_0$ at time 1 for sure.
- ▶ On the other hand, with probability p , the upward outcome is uS_0 , whereas the downward outcome dS_0 occurs with a probability of $1 - p$.
- ▶ Being risk neutral means that the expected return of the risky asset is the risk-free r_0 .

$$\frac{\mathbb{E}_0^Q(S_1)}{S_0} = e^{r_0} \implies uS_0 \times p + dS_0 \times (1 - p) = e^{r_0}S_0. \quad (4)$$

Risk-Neutral Probability

- Solving Equation (4) for p , we obtain the probability p of upward movement:

$$p = \frac{e^{r_0} - d}{u - d}. \quad (5)$$

- The probability q of downward movement is

$$q = 1 - p = \frac{u - e^{r_0}}{u - d}. \quad (6)$$

- Since the probability is positive, it must be that

$$d < e^{r_0} < u. \quad (7)$$

Numerical Illustration

- ▶ Suppose the up factor u is 1.25. That is 25% return.
- ▶ For the binomial tree to be recombining, the down factor is $d = 1/u = 1/1.25 = 0.80$, i.e., a decline of 20%.
- ▶ Let r_0 be 1%, hence $e^{0.01} = 1.01$.
- ▶ The risk-neutral probability of upward movement is

$$p = \frac{1.01 - 0.80}{1.25 - 0.80} = 46.67\%$$

- ▶ The risk-neutral probability of downward movement is

$$q = 1 - p = \frac{1.25 - 1.01}{1.25 - 0.80} = 53.33\%.$$

Risk-Neutral Probability in Call and Put Prices

▶▶▶▶ Applying the put-call parity, you can compute from (3) to obtain

$$p_0 = \frac{ue^{-r_0} - 1}{u - d}(K - dS_0). \quad (8)$$

▶▶▶▶ The risk-neutral probability (5) is embodied in the pricing formulas for call and put options, (3) and (8), respectively. To see it, we rewrite these pricing formulas as

$$c_0 = \frac{e^{r_0} - d}{u - d}e^{-r_0}(uS_0 - K), \quad p_0 = \frac{u - e^{r_0}}{u - d}e^{-r_0}(K - dS_0).$$

▶▶▶▶ It is easy to find that

$$c_0 = p e^{-r_0}(uS_0 - K), \quad p_0 = q e^{-r_0}(K - dS_0).$$

Numerical Examples

From the earlier example, we have $p = 46.67\%$, $q = 53.33\%$, $u = 1.25$, and $d = 0.80$. Also the risk-free rate is $r_0 = 1\%$.

Suppose $S_0 = \$5$, and the strike price K is $\$5.5$.

The call price is

$$c_0 = 0.4667 \times e^{-0.01} \times (1.25 \times \$5 - \$5.25) = \$0.46.$$

The put price is

$$p_0 = 0.5333 \times e^{-0.01} \times (\$5.25 - 0.8 \times \$5) = \$0.66.$$

Revisiting the First Principle of QF

- ▶ We denote the value of the call option at time 1 as $c_1^+ := uS_0 - K$ for the up state, and that for the down state as $c_1^- = 0$.
- ▶ In general c_1^- may not be zero when the options are deep in the money.
- ▶ With the risk-neutral probability p , we invoke the notion of expected future value of c_1 , which is denoted by $\mathbb{E}_0^{\mathbb{Q}}(c_1)$, and we write

$$c_0 = e^{-r_0}(pc_1^+ + (1-p)c_1^-) = e^{-r_0}\mathbb{E}_0^{\mathbb{Q}}(c_1). \quad (9)$$

$$p_0 = e^{-r_0}(pp_1^+ + (1-p)p_1^-) = e^{-r_0}\mathbb{E}_0^{\mathbb{Q}}(p_1). \quad (10)$$

- ▶ Under the risk-neutral probability p , the fair price of option today is the expected value of its future payoff discounted by the risk free rate.

Delta Hedging

- ➔ To make the portfolio value **invariant to both outcomes**, i.e., remove the uncertainty arising from the randomness of coin tossing, the long position in the stock is required to **hedge** against a short position in the call option.

➔ **Delta-hedging ratio**

$$x_0 = \frac{c_1^+ - c_1^-}{S_1^+ - S_1^-}, \quad (11)$$

where S_1^+ is the value S_1 of the underlying asset at time 1 in the up state and S_1^- is the value for the down state.

- ➔ Since $S_1^+ = uS_0$, $S_1^- = dS_0$, $c_1^+ = uS_0 - K$, and $c_1^- = 0$, the delta-hedging ratio (11) indeed yields the same result as (2):

$$x_0 = \frac{uS_0 - K}{(u - d)S_0}.$$

One-Period Change of Wealth

- ➔ The initial cash amount or wealth is denoted by W_0 . You buy x_0 shares of the underlying asset of the call option at the known price of S_0 . The left-over cash is

$$M_0 := W_0 - x_0 S_0, \quad (12)$$

which is invested in the risk-free money market.

- ➔ One period later, the wealth W_1 will become

$$W_1 = \begin{cases} W_1^+ = x_0 S_1^+ + e^{r_0} M_0, & \text{blue outcome;} \\ W_1^- = x_0 S_1^- + e^{r_0} M_0, & \text{red outcome.} \end{cases}$$

It is important to note that the two outcomes are **anticipated** at time 0.

Replication of Call's Payoff

- ➔ The replication approach is to make the cash flow W_1 at time 1 equal the option's payoff c_1 :

$$W_1 = x_0 S_1 + e^{r_0} M_0 = c_1.$$

- ➔ To achieve this replication, using definition (12), we first rewrite the cash flow W_1 of the portfolio as

$$\begin{aligned} W_1 &= e^{r_0} W_0 + x_0 (S_1 - e^{r_0} S_0) \\ &= e^{r_0} (W_0 + x_0 (e^{-r_0} S_1 - S_0)). \end{aligned}$$

- ➔ Replication: matching each of the possible outcome

$$\begin{aligned} W_0 + x_0 (e^{-r_0} S_1^+ - S_0) &= e^{-r_0} c_1^+, \\ W_0 + x_0 (e^{-r_0} S_1^- - S_0) &= e^{-r_0} c_1^-. \end{aligned}$$

Pricing by Replication

- ➔ We just need to solve for the values of x_0 and W_0 .
- ➔ Multiplying the upward outcome by p and the downward outcome by $1 - p$, and after adding them together, we obtain

$$W_0 + x_0 (e^{-r_0} (pS_1^+ + (1-p)S_1^-) - S_0) = e^{-r_0} (pc_1^+ + (1-p)c_1^-).$$

- ➔ Because of (4), the sum of the two terms, $pS_1^+ + (1-p)S_1^-$, is equal to $e^{r_0}S_0$.
- ➔ Consequently,

$$W_0 = e^{-r_0} (pc_1^+ + (1-p)c_1^-).$$

In view of (9), W_0 is in fact the value of the option c_0 at time 0.

Multi-Period Generalization

- ➔ For each node on the binomial tree that is not an ending node, the cash flow W_t at time t is

$$W_t = \begin{cases} W_t^+ = x_{t-1}S_t^+ + e^{r_0}M_{t-1}, & \text{blue outcome;} \\ W_t^- = x_{t-1}S_t^- + e^{r_0}M_{t-1}, & \text{red outcome.} \end{cases}$$

- ➔ The **money market account** M_t maturing at time $t + 1$ is

$$M_t = W_t - x_t S_t, \quad \text{for } t = 0, 1, 2, \dots, T - 1.$$

It is the fund left (or needed if M_t is negative) after taking a long position of x_t in the risky underlying asset at the price of S_t .

Multi-Period Generalization (cont'd)

➔ The **delta-hedging ratio** at time t for $t + 1$ is

$$x_t = \frac{c_{t+1}^+ - c_{t+1}^-}{S_{t+1}^+ - S_{t+1}^-} = \frac{c_{t+1}^+ - c_{t+1}^-}{(u - d)S_t}. \quad (13)$$

➔ Moreover, the risk-neutral pricing model is

$$c_t = e^{-r_0} \mathbb{E}_t(c_{t+1}). \quad (14)$$

Proposition

- $W_t = c_t$ from time 0 up to time $T - 1$.
- For each t of the binomial tree, the risk-neutral valuation of a pair of future payoffs is

$$c_t = e^{-r_0} (pc_{t+1}^+ + (1 - p)c_{t+1}^-) = e^{-r_0} \mathbb{E}(c_{t+1}). \quad (15)$$

Proof of Proposition by Induction

➔ Assume that $W_t = c_t$ is true and show that $W_{t+1} = c_{t+1}$ also holds.

➔ First, replication means that for the up state, i.e., $S_{t+1}^+ = uS_t$,

$$W_{t+1}^+ = x_t S_{t+1}^+ + e^{r_0} (W_t - x_t S_t) = e^{r_0} W_t + x_t S_t (u - e^{r_0}), \quad (16)$$

➔ Substituting the delta-hedging ratio x_t (13) into (16), we obtain

$$\begin{aligned} W_{t+1}^+ &= e^{r_0} W_t + \frac{(c_{t+1}^+ - c_{t+1}^-)(u - e^{r_0})}{u - d} \\ &= e^{r_0} W_t + (1 - p)c_{t+1}^+ - (1 - p)c_{t+1}^-. \end{aligned}$$

Proof of Proposition by Induction (cont'd)

- ➔ In view of (14) and the forward induction assumption that $W_t = c_t$, we have

$$e^{r_0} W_t = e^{r_0} c_t = \mathbb{E}_t^{\mathbb{Q}}(c_{t+1}) = pc_{t+1}^+ + (1-p)c_{t+1}^-.$$

Hence,

$$W_{t+1}^+ = pc_{t+1}^+ + (1-p)c_{t+1}^+ = c_{t+1}^+.$$

- ➔ Second, using the same method, you can also show that

$$W_{t+1}^- = pc_{t+1}^- + (1-p)c_{t+1}^- = c_{t+1}^-.$$

- ➔ Accordingly, if $W_t^{\pm} = c_t^{\pm}$, then $W_{t+1}^{\pm} = c_{t+1}^{\pm}$. We have already shown that $t = 0$ is true, i.e., $W_0 = c_0$. At time $t = 1$, $W_1 = c_1$ must also be true, and so on. Thus, the proof by forward induction is complete.

Connection of Volatility to the Up and Down Factors

- ➔ The up and down factors depend on the rate of variance σ^2 of the underlying asset's return.
- ➔ The rate of variance σ^2 quantifies the degree of fluctuation exhibited by the return on the underlying asset.
- ➔ The variance for a time period t is $\sigma^2 t$, and the **volatility** is its square root $\sigma\sqrt{t}$. We set

$$u = e^{\sigma\sqrt{t}}, \quad \text{and} \quad d = e^{-\sigma\sqrt{t}}.$$

- ➔ Note from (7) that the risk-free factor $e^{r_0 t}$ must be smaller than the up factor u , i.e., $e^{r_0 t} < e^{\sigma\sqrt{t}}$. It follows that the time interval t of each period must satisfy

$$\sqrt{t} < \frac{\sigma}{r_0}.$$

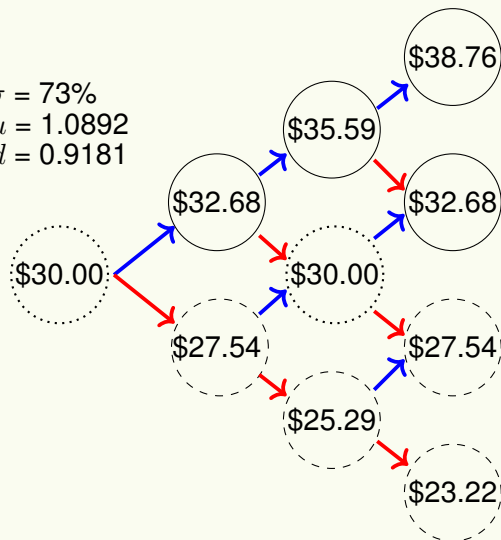
A Numerical Example of Binomial Option Pricing

- ➔ Asset prices for all nodes
- ➔ $S_0 = \underline{\hspace{2cm}}$
- ➔ Put option's days to maturity = 15 days
- ➔ Since $N = 3$, each period is $15/3 = 5$ days
- ➔ 5 days is $t = 5/365 = 1/73$ years
- ➔ risk-free rate $r_0 = 0.25\%$

$$\sigma = 73\%$$

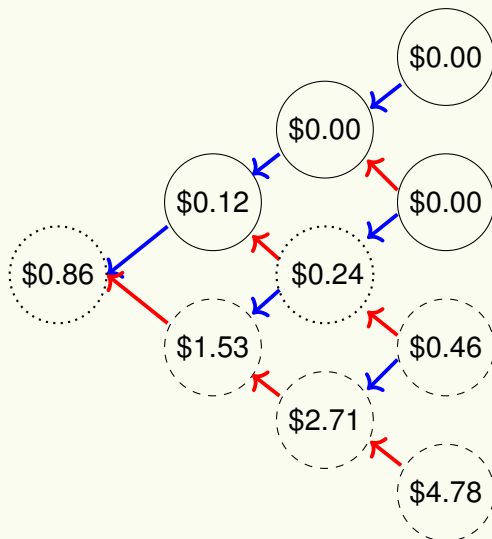
$$u = 1.0892$$

$$d = 0.9181$$



Put Option Prices

- ➔ Strike price = \$28
- ➔ Upward probability $p = 47.89\%$



Random Walk Once More

- The multiplicative random walk (1) in the context of multi-period binomial model takes the following form:

$$S_t = u^k d^{t-k} S_0.$$

- What is the probability of reaching $S_t = u^2 d S_0$?
- Answer: The binomial probability for the random number \tilde{N} of obtaining the blue dot on top when tossing the blue-red coins T times. In this case, $\tilde{N} = 2$ and $T = 3$. Therefore the probability is

$$\mathbb{P}(3; N = 2) = \binom{3}{2} p^2 (1-p)^1 = 3p^2(1-p).$$

Recall Your Pre-U Math

- In general, the binomial probability of the number of “successes” in flipping the blue-red coin, which turns up the blue dot is

$$\mathbb{P}(t; N = k) = \binom{t}{k} p^k (1 - p)^{t-k}, \quad (17)$$

where the **binomial coefficient** is

$$\binom{t}{k} := \frac{t!}{k!(t-k)!}.$$

- Here \tilde{N} is a random variable, as the number of successes is uncertain before the t tosses are completed.

Applying Your Pre-U's Binomial Theorem

- With the **probability mass function** (17) of the random variable \tilde{N} for the process of flipping the blue-red coin t times, we can compute the expected value at time 0 of \tilde{S}_t given S_0 as follows:

$$\mathbb{E}_0(S_t) = \sum_{k=0}^t u^k d^{t-k} S_0 \mathbb{P}(t; N = k).$$

- Interestingly, we find that

$$\begin{aligned} \mathbb{E}_0(S_t) &= \sum_{k=0}^t u^k d^{t-k} S_0 \binom{t}{k} p^k (1-p)^{t-k} \\ &= S_0 \sum_{k=0}^t \binom{t}{k} (up)^k (d(1-p))^{t-k} \\ &= S_0 (up + d(1-p))^t. \end{aligned}$$

First Principle of QF Once Again!

- Moreover, using the one-period risk-neutral probability p , i.e., (5), we obtain

$$\mathbb{E}_0^{\mathbb{Q}}(S_t) = S_0((u-d)p + d)^t = S_0 e^{r_0 t}.$$

The reason for using (5) is that each time period we consider here is one unit of time.

- To gain further insight, notice that

$$up + d(1-p) = (u-d)p + d = e^{r_0}$$

- Accordingly, under the single-period risk-neutral probability p in (5), the average gross return over one period is simply the **forward factor** e^{r_0} .
- Multi-period generalization

$$\boxed{S_0 = e^{-r_0 t} \mathbb{E}_0^{\mathbb{Q}}(S_t)}. \quad (18)$$

Large-Scale Binomial Probability

- Define a **probability mass function** $B(x)$ of a discrete variable x :

$$B(x) := \mathbb{P}(T; N = x) = \frac{T!}{x!(T-x)!} p^x (1-p)^{T-x}. \quad (19)$$

It is the binomial probability of x number of successes in getting the blue dot on top out of T tosses.

- The large number T is the result of slicing the time period t into many tiny pieces of size δt , which is a very short duration. We write

$$T = \frac{t}{\delta t}.$$

- It is noteworthy that T can be made arbitrarily large when δt is set at an arbitrarily small number. **Even so, their product, i.e., $T \times \delta t$ is a non-zero finite number t .**

Mean and Variance

- Under the risk-neutral probability p , the mean of the random variable A_i in (1) is, as computed before,

$$\mathbb{E}_0^{\mathbb{Q}}(A_i) = up + d(1 - p) = e^{r_0\delta t}, \quad \text{for each } i$$

- The variance of A_i is

$$\begin{aligned} \mathbb{V}^{\mathbb{Q}}(A_i) &= (u - e^{r_0\delta t})^2 p + (d - e^{r_0\delta t})^2 (1 - p) \\ &= p \left((u - e^{r_0\delta t})^2 - (d - e^{r_0\delta t})^2 \right) + (d - e^{r_0\delta t})^2 \\ &= (e^{r_0\delta t} - d)(u + d - 2e^{r_0\delta t}) + (e^{r_0\delta t} - d)^2 \\ &= p(1 - p)(u - d)^2. \end{aligned}$$

- Given that δt is small, the variance $\mathbb{V}(A_i)$ is well approximated by

$$\mathbb{V}^{\mathbb{Q}}(A_i) \approx 4p(1 - p)\sigma^2\delta t.$$

Takeaways

- Random walk, though simple, is a great model with which to think about randomness and probability.
- Binomial trees are useful models for pricing options.
- up factor, down factor, risk-free rate, and risk neutral probability are closely related.
- Up and down factors are dependent on the volatility of the underlying asset.
- In the asymptotic limit, the binomial model converges to the **Black-Scholes formula** for pricing European options.

Assignment

- 1 Given the parametrization $u = e^{\sigma\sqrt{t}}$ and risk-free rate r_0 for a recombining binomial tree, show that the risk-neutral upward probability p can be well approximated as

$$p = \frac{e^{r_0 t} - d}{u - d} \approx \frac{1}{2} \left(1 + \frac{(r_0 - \frac{1}{2}\sigma^2) \sqrt{t}}{\sigma} \right).$$

- 2 Given $t = 1/73$, $r_0 = 0.25\%$, and $\sigma = 73\%$ as in Slide 32, how good is the above approximation of p compared to the exact value in Slide 33?
- 3 Suppose $\delta t = t$, i.e., no splicing of the period. Using the same parameter values in Problem (2), compute the variance of A_i under the risk-neutral probability p .
- 4 Given the binomial price tree in Slide 32, price the ATM call option.

Additional Exercises

- For the one-dimensional random walk with p being the upward move by $+1$ and q being the downward move by -1 , what is the probability for the drunken man to be at $S_n = x > 0$ after n steps? (Hint: Let $f(m)$ be the probability that $x = m$ is ever reached. Then $f(m+1) = f(m)f(1)$.)
- For the one-dimensional random walk in Problem 1, Let $m > 0$ and $n > 0$. What is the probability $g(m, n)$ of reaching the point $x = +m$ before $x = -n$?
- A stock analyst with some special powers is able to guess correctly the flip of a coin with 60% probability. He starts with two million dollars, and plays a game of guessing the toss with a sovereign wealth fund, which has almost infinite amount of money. What is the probability that the stock analyst will ultimately lose all the money?