

Week 1: Preface

Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.

Albert Einstein

Motivated by the burning needs in the industry, applied mathematics, computing technologies, and finance coalesce and fuse into one interdisciplinary field called Quantitative Finance (QF). It overlaps *substantially* with Financial Engineering, which focuses on designing structured products as the name suggests, and with Financial Mathematics, which focuses on mathematical modeling, as well as with Computational Finance, which focuses on the efficient implementation of QF models on computers.

Perhaps a more succinct description of QF is “banking & finance through mathematical and computational models.” The application domains of QF are multifarious, encompassing pricing & hedging, risk management, and quantitative trading. QF takes the scientific approach to modeling and applies the models to generate profits, given that the accompanying risks are thoroughly analyzed and managed.

It goes without saying that in order to master QF, mathematical reasoning skills and programming proficiency are indispensable. This set of notes, which are synthesized from various sources, attempts to provide a basic grounding in mathematics that is needed to achieve mastery of Quantitative Finance major in an undergraduate program.

Commencing from set theory, the concepts of limits and continuity are discussed, along with calculus, probability, statistics, and stochastic processes. One caveat is that the mathematical rigor in these notes is by no means at the same level of what is normally expected of a math professor teaching math to the math students in the math department. That said, we take comfort in the fact that in physics, the math used is seldom rigorous, yet a plethora of natural phenomena can be described with amazing accuracy.

I prefer *germane rigor*, rather than being stifled by the rigor for the sake of rigor, which brings only marginal benefit to the logical tightness despite all the exacting pre-conditions and detour after detour to deal with pathological cases and generalizations that are void of concrete examples. At the end of the day, QF is an applied discipline and a language used by the “quants” in the marketplace. Intuitions derived from the math is much more valuable than rigor. But with appropriate rigor, QF produces workable solutions to generate, for example, an estimate for a derivative’s fair price that will prevent

risk-free arbitrage from happening. Combined with computing technologies, the buying and selling dynamics of a limit-order book can also be modeled in *real time*, even at the sub-millisecond resolution.

I hope that students read these preparatory notes with the above-mentioned motivations in mind. The mathematical topics and examples are chosen carefully and selectively to add value to the learning journey. Those that are of no or little relevance are omitted.

As much as possible, the notes also attempt to provide from first principle and axioms, direct logical sequence to an important result such as Taylor's expansion. Wherever appropriate, a sketch of the pathway to the result is provided at the beginning to provide extra motivation to carry on.

It should come as no surprise that the notes are far from perfect. Typos and errata inadvertently will arise. Moreover, inconsistencies in typesetting and the symbols used may make learning difficult. With best efforts given limited time, and knowing the learning inertia of reading thick books, the notes are kept concise. To make the learning experience interactive, the points in the notes are numbered so as to facilitate questions to be asked in a precise manner. The notes are also written for your reference in the future.

At the end of each note, there are a few questions for you to solve. They are purposed to enhance your conceptual understanding and to improve your mathematical skills. So do take about 1 hour or so to digest the points and then solve the exercise questions. Please also submit your solutions in a simple report format. The time is also ripe for you to learn how to write technical reports that require many formulas.

Finally, I wish you *bon voyage* in the learning journey!

Christopher Ting