

Lesson 14 Tutorial Questions

14.1 Find the matrix \mathbf{P} that diagonalizes $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$.

14.2 Let $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$. For what values of a is $\mathbf{u} = \begin{bmatrix} a \\ -1 \end{bmatrix}$ an eigenvector of \mathbf{A} ?

14.3 Let $\mathbf{A} = \begin{bmatrix} 3 & -6 & 0 \\ -6 & 0 & 6 \\ 0 & 6 & -3 \end{bmatrix}$.

- What are the eigenvalues of \mathbf{A} ?
- Find all the normalized eigenvectors (norm = 1) corresponding to the eigenvalues.
- What is the matrix \mathbf{P} that diagonalizes \mathbf{A} ?

14.4 Let $\mathbf{A} = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 3 & a \\ 4 & -2 & -1 \end{bmatrix}$ for some variable a . Find all values of a which will guarantee that \mathbf{A} has eigenvalues 0, 3, and -3.

14.5 Suppose that \mathbf{A} is a diagonalizable matrix with the characteristic polynomial:

$$\Phi_{\mathbf{A}}(\lambda) = \lambda^2(\lambda - 3)(\lambda + 2)^3(\lambda - 4)^3.$$

- What is the dimension of the square matrix \mathbf{A} ?
- What is the dimension of the eigenspace corresponding to the eigenvalue $\lambda = 4$?
- What is the dimension of the kernel (nullspace) of \mathbf{A} ?

14.6 Let $\mathbf{A} = \begin{bmatrix} 10 - 5a & -16 + 10a \\ 5 - 3a & -8 + 6a \end{bmatrix}$. What should a be so that the matrix \mathbf{A} is *not* diagonalizable?