Lesson 10 Tutorial Questions

10.1 The dimension of the kernel of a map is zero.
Is the map injective? Note: You have to justify or explain your answer.

10.2 Let $T : \Re^2 \to \Re^3$ be a transformation such that $T(\boldsymbol{e}_1) = \boldsymbol{u}_1$ and $T(\boldsymbol{e}_2) = \boldsymbol{u}_2$, where $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are unit vectors of $\Re^2$ and

$$\mathbf{u}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

(a) Is the map linear?
Note: You have to justify or explain your answer.

(b) Find $T\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)$.

10.3 Suppose that a real-valued matrix $\boldsymbol{A}$ transforms each of the following vectors

$$\boldsymbol{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

into the vectors, respectively,

$$\boldsymbol{y}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \boldsymbol{y}_2 = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \boldsymbol{y}_3 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

That is, $\boldsymbol{A}\boldsymbol{x}_i = \boldsymbol{y}_i$ for $i = 1, 2, 3$. Find the matrix $\boldsymbol{A}$.

10.4 Define two maps $T : \Re^2 \to \Re^2$ and $S : \Re^2 \to \Re^2$ by, respectively,

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + y \\ 0 \end{bmatrix} \quad \text{and} \quad S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ xy \end{bmatrix}.$$

(a) Is T a linear transformation?

(b) Is S a linear transformation?

(c) is $S \circ T$ a linear transformation?

10.5 Let P_3 be the vector space of all polynomials of degree 3 or less with real coefficients. Suppose $T : P_3 \to V$ is the linear transformation defined by

$$T\left(a_0 + a_1 x + a_2 x^2 + a_3 x^3\right) = \begin{bmatrix} a_0 + a_2 & -a_0 + a_3 \\ a_1 - a_2 & -a_1 - a_3 \end{bmatrix}$$

for any polynomial $a_0 + a_1 x + a_2 x^2 + a_3 x^3 \in P_3$, and V is the range of T.

(a) What is $T\left(2 + 3x^2\right)$?

(b) Find a basis consisting of 2×2 matrices for the range of T that corresponds to the basis of the domain P_3, namely, $\left\{1, x, x^2, x^3\right\}$.

(c) What is the rank of T?

$$\left(\text{Hint: The representation matrix of the transformation } T \text{ is } \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}.\right)$$