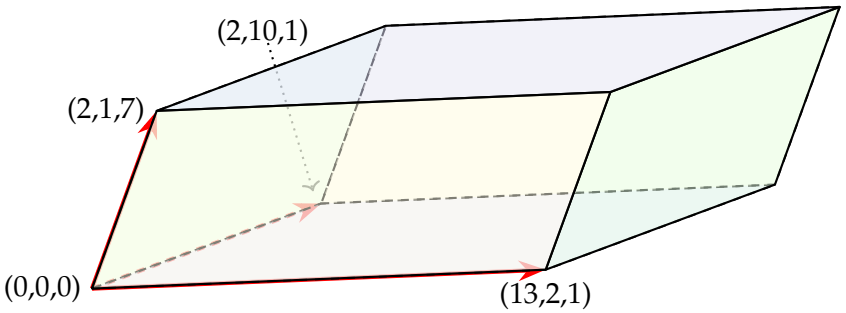
Lesson 6 Tutorial Questions

6.1 Consider a parallelepiped in a Cartesian frame of coordinates (x, y, z) in meters.. The three vertices from the origin (0,0,0) are indicated as follows:



What is the volume of this parallelepiped in cubic meters?

6.2 Consider a system of linear equations: $\begin{cases} 6x - 7y = b_1 \\ 5x + 6y = b_2 \end{cases}$. Use Theorem 3.8 to determine whether the inverse of the coefficient matrix exists.

6.3 Is the system of equations in Question 6.2 solvable. If not, explain why. If yes, apply Cramer's formula to find the solutions.

6.4 Find the determinant of the matrix: $\boldsymbol{A} = \begin{bmatrix} 100 & 101 & 102 \\ 101 & 102 & 103 \\ 102 & 103 & 104 \end{bmatrix}$.

6.5 Let $\boldsymbol{A} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$. Notice that $\boldsymbol{A}$ contains every integer from 1 to 9 and that the sums of each row, column, and diagonal of A are equal. Such a grid is sometimes called a magic square. Compute the determinant of $\boldsymbol{A}$.

6.6 Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be $n \times n$ matrices, where n is an integer greater than 1. Is it true that

$$\det(\boldsymbol{A} + \boldsymbol{B}) = \det(\boldsymbol{A}) + \det(\boldsymbol{B})?$$

If so, then give a proof. If not, then give a counterexample.

6.7 Find all values of x such that $\boldsymbol{A}$ is invertible.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 0 & 10 \\ 0 & 7+x & -3 \\ 0 & 4 & x \end{bmatrix}.$$

6.8 Let $\boldsymbol{A} = \begin{bmatrix} 1 & -x & 0 & 0 \\ 0 & 1 & -x & 0 \\ 0 & 0 & 1 & -x \\ 0 & 1 & 0 & -1 \end{bmatrix}$. Find all values of x so that the matrix $\boldsymbol{A}$ is singular (i.e., not invertible).

6.9 If the inverse of a square matrix $\boldsymbol{A}$ is equal to itself, i.e., $\boldsymbol{A}^{-1} = \boldsymbol{A}$, what then is the determinant of $\boldsymbol{A}$? (Hint: Theorem 4.6 (Inverse of a Matrix))

6.10 Consider a 4-dimensional matrix $\boldsymbol{M}$:

$$\boldsymbol{M} = \begin{bmatrix} 2 & 5 & 3 & -5 \\ 1 & -3 & 2 & 1 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & -6 & 2 \end{bmatrix}$$

What is the determinant of $\boldsymbol{M}$?
(Hint: Apply Theorem 3.11.)