

Lesson 2 Tutorial Questions

2.1 Which one of the following types of square matrix is *not* equal to its own transpose?

- (a) An identity matrix
 (b) A diagonal matrix
 (c) A triangular matrix
 (d) A scalar matrix

2.2 When a square matrix is added to its own transpose, the resulting matrix is

- (a) an identity matrix.
 (b) a symmetric matrix.
 (c) a zero matrix.
 (d) a triangular matrix.

2.3 Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{bmatrix}$.

- (i) Is the subtraction $\mathbf{A} - \mathbf{B}$ possible?
 (ii) What is the transpose of \mathbf{C} ?
 (iii) Is the product $\mathbf{B}'\mathbf{A}$ possible under matrix multiplication?
 (iv) Suppose \mathbf{A} is expressed as $[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{a}_4]$, and \mathbf{B} as $[\mathbf{b}_1 \ \mathbf{b}_2]$. Find $\mathbf{a}_3 + \mathbf{b}_2$.
 (v) What is the trace of matrix $\mathbf{D} := \mathbf{BC}$?

2.4 Amy speaks French and German; Ben speaks English, French and Hebrew; Charles speaks English, Hebrew, and Italian; Dan speaks all the languages the others speak except French; and no one speaks any other language. Construct a matrix $\mathbf{A} = [a_{ij}]$ with rows representing the four people mentioned and columns representing the languages they speak. In alphabetical order, put $a_{ij} = 1$ if person i speaks language j and $a_{ij} = 0$ otherwise.

- (i) What is \mathbf{A} according to the specification?
 (ii) Compute \mathbf{AA}' and then interpret the results.

2.5 What is the inverse of matrix $\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$?