Sample Tutorial Questions

1.1 A regular matrix is

(a) an identity matrix.
(b) a symmetric matrix.
(c) a matrix where all the entries are 1.
(d) an invertible matrix.

1.2 If $\boldsymbol{M}$ is a 2×2 scalar matrix, then (if it exists) the inverse of $\boldsymbol{M}$, i.e., $\boldsymbol{M}^{-1}$, is

(a) the identity matrix.
(b) a scalar matrix.
(c) a lower triangular matrix.
(d) an upper triangular matrix.

1.3 Let $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{bmatrix}$, and $\boldsymbol{C} = \begin{bmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{bmatrix}$.

(i) Is the addition $\boldsymbol{A} + \boldsymbol{B}$ possible?
(ii) What is the transpose of $\boldsymbol{B}$?
(iii) Is the product $\boldsymbol{C}'\boldsymbol{A}$ possible under matrix multiplication?
(iv) Suppose $\boldsymbol{A}$ is expressed as $\begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{a}_4 \end{bmatrix}$, and $\boldsymbol{B}$ as $\begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 \end{bmatrix}$. Find $\boldsymbol{a}_4 + \boldsymbol{b}_1$.
(v) What is the trace of matrix $\boldsymbol{D} := \boldsymbol{C}\boldsymbol{B}$?

1.4 Amy speaks French and German; Ben speaks English, French, and Hebrew; Charles speaks English, Hebrew, and Italian; Dan speaks all the languages the others speak except French; and no one speaks any other language. Construct a matrix $\boldsymbol{A} = [a_{ij}]$ with rows representing the four people mentioned and columns representing the languages they speak. In alphabetical order, put $a_{ij} = 1$ if person i speaks language j and $a_{ij} = 0$ otherwise.

(i) What is $\boldsymbol{A}$ according to the specification?
(ii) Compute $\boldsymbol{A}'\boldsymbol{A}$ and then interpret the results.

1.5 What is the inverse of matrix $\boldsymbol{A} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$?