Outlines of Lesson 13

0. Quick Revision of Previous Lesson

0.1 Feedback on your Lesson 11 quiz answers

1. Introduction

1.1 Why do we need to study eigenvalue and eigenvector?

1.2 Why does a distinguished computational biologist love linear algebra, probability and statistics?

1.3 What is the intuitive meaning of eigenvalue?

1.4 What is the intuitive meaning of eigenvector?

2. Eigenvalue and Eigenvector

2.1 What is eigenvalue, eigenvector, and eigenspace?

2.2 What is the role of determinant in the study of eigenvalue and eigenvector?

2.3 What is the eigen equation of a square matrix

2.4 What is the multiplicity of a eigenvalue?

2.5 What is the eigen polynomial of a transformation?

2.6 How is polynomial equation connected to Lesson 3?

3. Properties of Eigenvalues

3.1 What are the properties of an eigen polynomial?

3.2 What is the dimension of eigenspace?

3.3 Are the eigenvectors linearly independent?

4. Diagonalization and Triangularization

4.1 What is the role of eigenvectors in the diagonalization of a matrix?

4.2 What are the conditions for a matrix to be diagonalizable?

4.3 What is the connection between the trace of a matrix and eigenvalues?

4.4 What is the conditions for a matrix to be diagonalizable?

4.5 What is the connection between symmetric matrix and diagonalization?

Quiz questions of Lesson 13

1 Name one of the 4 pillars of machine learning application in Slide 4.

Answer: ______________________

2 The eigenvector $\boldsymbol{v}$ of an eigenvalue of $\boldsymbol{A}$ is unique. True or false?

Answer: ______________________

3 What is the highest order of the eigen polynomial of a 5-dimensional square matrix?

Answer: ______________________

4 Let $\boldsymbol{A} = \begin{bmatrix} 5 & -2 \\ 6 & -2 \end{bmatrix}$ and $\boldsymbol{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Compute the following:

(a) $\boldsymbol{Ax}$ (b) $\boldsymbol{A}^2\boldsymbol{x}$ (c) $\boldsymbol{A}^{2020}\boldsymbol{x}$

Answer: ______________________

5 If $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ is an eigenvector, then $\begin{bmatrix} -14 \\ -6 \end{bmatrix}$ is also an eigenvector that corresponds to the same eigenvalue. True or false?

Answer: ______________________

6 Let $\boldsymbol{M} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. What is the eigenvalue of $\boldsymbol{M}$?

Answer: ______________________

7 What is the multiplicity of the eigenvalue of Question 6?

Answer: ______________________

8 Continuing from Question 6, what is the eigenvector such that its norm is 1?

Answer: ______________________

9 With respect to the 3-dimensional space, $\mathbb{R}[x]_2 = \{a_0 + a_1 x + a_2 x^2 | a_0, a_1, a_2 \in \mathfrak{R}\}$, find the eigenvalues for the following linear transformation $T : \mathbb{R}[x]_2 \longrightarrow \mathbb{R}[x]_2$:

$$T\left(f(x)\right) = f''(x) + 2xf'(x) + 2f(x).$$

What is the sum of the representation matrix's eigenvalues?

Answer: ______________________

10 If the n eigenvalues of an n-dimensional square matrix $\boldsymbol{A}$ are all different, then $\boldsymbol{A}$ is not diagonalizable. True or false?

Answer: ______________________