Outlines of Lesson 11

0. Quick Revision of Previous Lesson

0.1 Feedback on your Lesson 9 quiz answers

1. Introduction

1.1 Why do we need to study inner product space?

1.2 What is the quote saying?

2. Inner Product

2.1 What is the definition of inner product?

2.2 What is the inner product space and norm?

2.3 What is so special about the standard inner product?

2.4 What is so special about the inner product of functions?

2.5 What is the intuitive meaning of the Cauchy–Schwarz inequality?

2.6 What is the intuitive meaning of the triangular inequality?

3. Orthogonal System

3.1 How is an orthogonal system defined?

3.2 What are the examples of an orthogonal system?

3.3 How do you prove that orthogonality implies linear independence?

3.4 What is an orthonormal system and an orthnormal basis?

3.5 What is an example of an orthnormal basis?

3.6 How is the orthonormal basis defined?

3.7 What is the intuitive meaning of the Gram-Schmidt process?

3.8 What is QR decomposition?

3.9 How is the Gram-Schmidt process applied to the Legendre polynomial and the Hermite polynomial?

3.10 What is the orthogonal complement space?

4. Orthogonal Matrix

4.1 What is the meaning of an orthogonal matrix?

4.2 What is orthogonal transformation?

4.3 How is orthogonal matrix connected to orthogonal transformation?

Quiz questions of Lesson 11

1 Let s and t be real numbers. Suppose $\boldsymbol{a}$ and $\boldsymbol{b}$ are the vectors of a linear space V. Which of the following is correct?

(a) $\langle s\boldsymbol{a}, t\boldsymbol{b}\rangle = s\langle \boldsymbol{a}, \boldsymbol{b}\rangle$ (b) $\langle s\boldsymbol{a}, t\boldsymbol{b}\rangle = t\langle \boldsymbol{a}, \boldsymbol{b}\rangle$ (c) $\langle s\boldsymbol{a}, t\boldsymbol{b}\rangle = st\langle t\boldsymbol{a}, s\boldsymbol{b}\rangle$ (d) $\langle s\boldsymbol{a}, t\boldsymbol{b}\rangle = \langle t\boldsymbol{a}, s\boldsymbol{b}\rangle$

Answer: _____

2 The inner product is defined for geometric vectors only. True or false?

Answer: _____

3 Suppose $\boldsymbol{a}' = \begin{bmatrix} -1 & -2 & 2 \end{bmatrix}$ belongs to an inner product space. What is the norm of $\boldsymbol{a}$?

Answer: _____

4 Standard inner product is a bi-linear map from $\Re^2$ to the interval $[0, \infty)$. True or false?

Answer: _____

5 Let $C([-1,1])$ be the linear space of all real-valued functions that are continuous over the interval $[-1,1]$. What is the value of the inner product of $f(x) = x + 3$ and $g(x) = 3x^2$?

Answer: _____

6 Orthogonality implies linear dependence. True or false?

Answer: _____

7 Which is an orthonormal basis?

(a) $\left\{ \cos(kx), \sin(kx) \right\}_{k=1}^{10} \cup \{1\}$

(b) $\left\{ \frac{\cos(kx)}{\sqrt{\pi}}, \frac{\sin(kx)}{\sqrt{\pi}} \right\}_{k=1}^{10} \cup \left\{ \frac{1}{\sqrt{\pi}} \right\}$

(c) $\left\{ \frac{\cos(kx)}{\sqrt{\pi}}, \frac{\sin(kx)}{\sqrt{\pi}} \right\}_{k=1}^{10} \cup \left\{ \frac{1}{\sqrt{2\pi}} \right\}$

(d) $\left\{ \cos(kx), \sin(kx) \right\}_{k=1}^{10}$

Answer: _____

8 The Gram-Schmidt process produces linearly independent vectors. True or false?

Answer: _____

9 In the QR decomposition of a matrix, what is the name of the R matrix?
(a) upper triangular matrix (b) lower triangular matrix (c) diagonal matrix (d) off-diagonal matrix

Answer: _____

10 What kind of matrix *must* an orthogonal matrix be?
(a) diagonal (b) off-diagonal (c) symmetric (d) regular

Answer: _____