Factor Models and Analysis

Christopher Ting

http://cting.x10host.com/CUHKSZ/CUHKSZ.html

http://www.mysmu.edu/faculty/christophert/
✉: christophert@smu.edu.sg

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Broad Lesson Plan

What is Factor Analysis?

- Are there relationships among many dependent variables?
- How many different factors are needed to explain the pattern of relationships among these variables?
- What is the nature of these factors?
- How well do the hypothesized factors explain the observed data?
- How much purely random or unique variance does each observed variable include?
- Goal: To discover something about the nature of the independent variables that affect dependent variables, even though those independent variables are not measured directly.

Functions of Factor Models

- Decompose risk and return into explainable and unexplainable components
- Generate estimates of abnormal return
- Describe the covariance structure of returns
- Predict returns in specified stress scenarios
- Provide a framework for portfolio risk analysis

Three Types of Factor Models

- Macroeconomic factor model
  - Factors are observable economic and financial time series.
- Fundamental factor model
  - Factors are created from observable asset characteristics.
- Statistical factor model (PCA)
  - Factors are unobservable and extracted from asset returns.

General Form

- R_{it}: simple return on asset i $(i = 1, ..., N)$ in time period t $(t = 1, \ldots, T)$.
- f_{kt}: k-th **common factors** $(k = 1, \ldots, M)$
- β_{ki}: **Factor loading** or **factor beta** for asset i on the k-th factor
- ϵ_{it}: asset's **specific factor**

$$\begin{aligned} R_{it} &= \alpha_i + \beta_{1i} f_{1t} + \beta_{2i} f_{2t} + \cdots + \beta_{Ki} f_{Mt} + \epsilon_{it} \\ &= \alpha_i + \boldsymbol{\beta}_i' \boldsymbol{f}_t + \epsilon_{it} \end{aligned} \tag{1}$$

Assumptions

- The factor realizations, $\boldsymbol{f}_t$, are stationary with unconditional moments

$$\mathbb{E}\left(\boldsymbol{f}_t\right) = \boldsymbol{\mu}_f$$

$$\mathbb{V}\left(\boldsymbol{f}_t\right) = \boldsymbol{\Omega}_f = \mathbb{E}\left((\boldsymbol{f}_t - \boldsymbol{\mu}_f)(\boldsymbol{f}_t - \boldsymbol{\mu}_f)'\right)$$

- Asset specific error terms, ϵ_{it}, are uncorrelated with each of the common factors, f_{kt},

$$\mathbb{C}(f_{kt}, \epsilon_{it}) = 0, \quad \text{for all } k, i, \text{ and } t.$$

- Error terms ϵ_{it} are serially uncorrelated and contemporaneously uncorrelated across assets

$$\mathbb{C}(\epsilon_{it}, \epsilon_{js}) = \begin{cases} \sigma_i^2, & \text{for all } i = j \text{ and } t = s; \\ 0, & \text{otherwise.} \end{cases}$$

Notations

- Vectors with a subscript t: **cross-section** of all assets

$$\boldsymbol{R}_t = \begin{bmatrix} R_{1t} \\ \vdots \\ R_{Nt} \end{bmatrix}$$

- Vectors with a subscript i: **time series** of a given asset

$$\boldsymbol{R}_i = \begin{bmatrix} R_{i1} \\ \vdots \\ R_{iT} \end{bmatrix}$$

- $T \times N$ matrix of all assets over all time periods

$$\boldsymbol{R} = \begin{bmatrix} R_{11} & \cdots & R_{N1} \\ \vdots & \ddots & \vdots \\ R_{1T} & \cdots & R_{NT} \end{bmatrix}$$

Alpha

- Expected return of asset i

$$\mathbb{E}\left(R_{it}\right) = \alpha_i + \boldsymbol{\beta}_i' \, \mathbb{E}(\boldsymbol{f}_t)$$

- $\boldsymbol{\beta}_i' \, \mathbb{E}(\boldsymbol{f}_t)$ = explained expected return due to systematic risk factors
- $\alpha_i =: \mathbb{E}(R_{it}) - \boldsymbol{\beta}_i' \, \mathbb{E}(\boldsymbol{f}_t)$ is he unexplained expected return.
- Equilibrium asset pricing models impose the restriction $\alpha_i = 0$ (no abnormal return) for all assets $i = 1, \ldots, N$.

Covariance

- Under cross-section regression,

$$\boldsymbol{R}_t = \boldsymbol{\alpha} + \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{\epsilon}_t, \quad t = 1, 2, \ldots, T,$$

  where $\boldsymbol{R}_t$ is a $N \times 1$ vector, and $\boldsymbol{B}$ is a $N \times M$ matrix.
- The $N \times N$ covariance matrix under the assumptions of multi-factor model is,

$$\mathbb{C}(\boldsymbol{R}_t) = \boldsymbol{B}\boldsymbol{\Omega}_f\boldsymbol{B}' + \boldsymbol{D},$$

  where the diagonal matrix is

$$\boldsymbol{D} := \mathbb{E}\left(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}' \middle| \boldsymbol{f}_t\right).$$

- That is, with $\sigma_i^2 := D_{ii}$,

$$\mathbb{V}(R_{it}) = \boldsymbol{\beta}_i'\boldsymbol{\Omega}_f\boldsymbol{\beta}_i + \sigma_i^2$$
$$\mathbb{C}(R_{it}, R_{jt}) = \boldsymbol{\beta}_i'\boldsymbol{\Omega}_f\boldsymbol{\beta}_j$$

An Example

- Consider a multi-factor of price change of an asset:

$$\Delta p_i = \mu_i + \beta_{i1}\Delta f_1 + \cdots + \beta_{iM}\Delta f_M + \epsilon_i.$$

- In this model
  - μ_i: mean price change of the i-th asset
  - Δf_j: change of the j-th factor
  - β_{ij}: factor loading of the i-th asset on the j-th factor
  - ϵ_i: idiosyncratic error of the i-th asset

- In vector-matrix form,

$$\Delta \boldsymbol{p} = \boldsymbol{\mu} + \boldsymbol{B}\Delta \boldsymbol{f} + \boldsymbol{\epsilon}.$$

Covariance of the Example

- We can relate the covariance matrix $\mathbf{\Omega}$ of the individual asset price changes to the covariance matrix $\mathbf{\Omega}_f$ of the factor changes:

$$
\begin{aligned}
\mathbf{\Omega} &= \mathbb{E}\left((\Delta \boldsymbol{p} - \boldsymbol{\mu})(\Delta \boldsymbol{p} - \boldsymbol{\mu})'\right) \\
&= \mathbb{E}\left((\boldsymbol{B}\Delta \boldsymbol{f} + \boldsymbol{\epsilon})(\boldsymbol{B}\Delta \boldsymbol{f} + \boldsymbol{\epsilon})'\right) \\
&= \boldsymbol{B}\,\mathbb{E}\left(\Delta \boldsymbol{f}\Delta \boldsymbol{f}'\right)\boldsymbol{B}' + \boldsymbol{B}\,\mathbb{E}\left(\Delta \boldsymbol{f}\boldsymbol{\epsilon}'\right) + \mathbb{E}\left(\boldsymbol{\epsilon}\Delta \boldsymbol{f}'\right)\boldsymbol{B}' + \mathbb{E}\left(\boldsymbol{\epsilon}\boldsymbol{\epsilon}'\right) \\
&= \boldsymbol{B}\,\mathbb{E}\left(\Delta \boldsymbol{f}\Delta \boldsymbol{f}'\right)\boldsymbol{B}' + \boldsymbol{D} \\
&= \boldsymbol{B}\mathbf{\Omega}_f\boldsymbol{B}' + \boldsymbol{D}
\end{aligned}
$$

Portfolio Analysis

- Let $\boldsymbol{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}$ be a vector of portfolio weights.
- Hence, given N simple returns R_{it}, the return of the portfolio, R_{pt}, is

$$R_{pt} = \boldsymbol{w}'\boldsymbol{R}_t = \sum_{i=1}^{N} w_i R_{it}.$$

- Portfolio Factor Model: Since $\boldsymbol{R}_t = \boldsymbol{\alpha} + \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{\epsilon}_t$, we get

$$R_{pt} = \boldsymbol{w}'\boldsymbol{\alpha} + \boldsymbol{w}'\boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{w}'\boldsymbol{\epsilon}_t =: \alpha_p + \boldsymbol{\beta}_p'\boldsymbol{f}_t + \epsilon_{pt}.$$

- The variance of R_{pt} is

$$\mathbb{V}(R_{pt}) = \boldsymbol{\beta}_p'\boldsymbol{\Omega}_f\boldsymbol{\beta}_p + \mathbb{V}(\epsilon_{pt}) = \boldsymbol{w}'\boldsymbol{B}\boldsymbol{\Omega}_f\boldsymbol{B}'\boldsymbol{w} + \boldsymbol{w}'\boldsymbol{D}\boldsymbol{w}.$$

Active versus Static Portfolio

- Active portfolios have weights that change over time due to active asset allocation decisions.
- Static portfolios have weights that are fixed over time (e.g. equally weighted portfolio).
- Factor models can be used to analyze the risk of both active and static portfolios.

Econometric Problems

- The model is
$$R_{it} = \alpha_i + \boldsymbol{\beta}_i' \boldsymbol{f}_t + \epsilon_{it}.$$
- Choose factors $\boldsymbol{f}_t$ (observed economic and financial time series).
- Estimate factor betas, $\boldsymbol{\beta}_i$, and residual variances, σ_i^2, using time series regression techniques.
- Estimate factor covariance matrix, $\boldsymbol{\Omega}_f$, from observed history of factors.

Sharpe's Single Factor Model

- Let R_{mt} denote the return or excess return (relative to the risk-free rate) on a market portfolio at time t.
- A macroeconomic factor model with a single market factor:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it},$$

for $i = 1, 2, \ldots, N$, and $t = 1, 2, \ldots, T$.

- **Risk-adusted expected return** and **abnormal return**

$$\mathbb{E}\left(R_{it}\right) = \beta_i \, \mathbb{E}\left(R_{mt}\right)$$

$$\alpha_i := \mathbb{E}\left(R_{it}\right) - \beta_i \, \mathbb{E}\left(R_{mt}\right)$$

Estimation of Sharpe's Single Factor Model

- Because R_{mt} is observable, the parameters β_i and σ_i^2 of the single factor model for each asset can be estimated using time series regression (i.e.,ordinary least squares).
- The "slope" estimate is, for $i = 1, 2, \ldots, N$,

$$\widehat{\beta}_i = \frac{\widehat{\sigma}_{im}}{\widehat{\sigma}_m^2}.$$

- The "intercept" estimate is, for $i = 1, 2, \ldots, N$,

$$\widehat{\alpha}_i = \overline{R}_i - \widehat{\beta}_i \overline{R}_m.$$

- The variance of the residuals is

$$\widehat{\sigma}_i^2 = \frac{1}{T-2} \widehat{\boldsymbol{\epsilon}}_i' . \widehat{\boldsymbol{\epsilon}}_i.$$

General Multi-Factor Model

- Model specifies M observable macro-variables

$$R_{it} = \alpha_i + \boldsymbol{\beta}_i' \boldsymbol{f}_t + \epsilon_{it}.$$

- Sometimes the macroeconomic factors are standardized to have mean zero and a common scale.
- The factors must be stationary (not trending).
- Sometimes the factors are made orthogonal.

Estimation: Multi-Factor Model

- The factor realizations $\boldsymbol{f}_t$ are observable and we can organize it as

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{f}_1' \\ \boldsymbol{f}_2' \\ \vdots \\ \boldsymbol{f}_T' \end{bmatrix}.$$

- For multi-factor model with a factor matrix $\boldsymbol{F}$, it is

$$\boldsymbol{R}_i = \alpha_i \mathbf{1}_T + \boldsymbol{F}\boldsymbol{\beta}_i + \boldsymbol{\epsilon_i}, \quad \text{for } i = 1, 2, \ldots, N.$$

- We define $\boldsymbol{X} := \begin{bmatrix} \mathbf{1}_T & \boldsymbol{F} \end{bmatrix}$.
- The factor loading is solved by OLS to yield, for each i,

$$\begin{pmatrix} \widehat{\alpha}_i & \widehat{\boldsymbol{\beta}}_i' \end{pmatrix}' = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}_i.$$

Estimation: Multi-Factor Model (Cont'd)

- The variance of the residuals is, by setting $K := M + 1$,

$$\widehat{\sigma}_i^2 = \frac{1}{T-K}\widehat{\boldsymbol{\epsilon}}_i'\widehat{\boldsymbol{\epsilon}}_i.$$

- The covariance matrix of the factor realizations may be estimated using the time series sample covariance matrix

$$\widehat{\boldsymbol{\Omega}}_f = \frac{1}{T-1}\sum_{t=1}^{T}\big(\boldsymbol{f}_t - \overline{\boldsymbol{f}}\big)\big(\boldsymbol{f}_t - \overline{\boldsymbol{f}}\big)', \quad \text{where } \overline{\boldsymbol{f}} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{f}_t.$$

- The estimated multi-factor model covariance matrix is then

$$\widehat{\boldsymbol{\Omega}}_{\mathrm{FM}} = \widehat{\boldsymbol{B}}\widehat{\boldsymbol{\Omega}}_f\widehat{\boldsymbol{B}}' + \widehat{\boldsymbol{D}}.$$

What is a Fundamental Factor Model?

- Fundamental factor models use observable asset specific characteristics (fundamentals) like industry classification, market capitalization, style classification (value, growth) etc. to determine the common risk factors.
- Factor betas are constructed from observable asset characteristics (i.e., $\boldsymbol{B}$ is known)
- Factor realizations, $\boldsymbol{f}_t$, are estimated/constructed for each t given $\boldsymbol{B}$.
- In practice, fundamental factor models are estimated in two ways.
    - BARRA approach
    - Fama-French Approach

BARRA Approach

- Pioneered by Bar Rosenberg, founder of BARRA Inc, which was merged with **MSCI** in 2004.
- Observable asset specific fundamentals (or some transformation of them) are treated as the factor betas, $\boldsymbol{\beta}_i$.
- Factor realizations at time t, $\boldsymbol{f}_t$, are treated as unobservable.
- The econometric problem is then to estimate the factor realizations at time t given the factor betas.
- Estimation is done by running T cross-section regressions.

BARRA-Type Single Factor Model

- Consider a single factor model in the form of a cross-sectional regression at time t:

$$\boldsymbol{R}_t = \boldsymbol{\beta} f_t + \boldsymbol{\epsilon}_t, \qquad t = 1, 2, \ldots, T.$$

- $\boldsymbol{\beta}$ is an $N \times 1$ vector of observed values of an asset specific attribute (e.g., market capitalization, industry classification, style classification).
- f_t is an unobserved factor realization.
- Assumptions include
  - $\mathbb{V}(f_t) = \sigma_f^2$
  - $\mathbb{C}(f_t, \epsilon_{it}) = 0$, for all $i = 1, 2, \ldots, N$.
  - $\mathbb{V}(\epsilon_{it}) = \sigma_i^2$, for all $i = 1, 2, \ldots, N$.

Estimation: BARRA Approch

- For each time period $t = 1, 2, \dots, T$, the vector of factor betas, $\boldsymbol{\beta}$, is treated as data and the factor realization f_t, is the parameter to be estimated.
- Since the error term $\boldsymbol{\epsilon}_t$ is heteroskedastic, efficient estimation of f_t is done by weighted least squares (WLS), assuming the asset specific variances σ_i^2 are known.

$$\widehat{f}_{t,\text{wls}} = (\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{R}_t, \quad t = 1, 2, \dots, T,$$

where $\boldsymbol{D}$ is a diagonal matrix with $\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2$ along the diagonal.

- σ_i^2 can be consistently estimated and a feasible WLS estimate can be computed.

Factor Mimicking Portfolio

- f_t in the BARRA approach has an interesting interpretation as the return on a portfolio $\boldsymbol{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_N \end{bmatrix}'$ that solves

$$\min_{\boldsymbol{h}} \boldsymbol{h}'\boldsymbol{D}\boldsymbol{h} \qquad \text{subject to } \boldsymbol{h}'\boldsymbol{\beta} = 1.$$

- The portfolio $\boldsymbol{h}$ minimizes asset return residual variance subject to having unit exposure to the attribute $\boldsymbol{\beta}$ and is given by

$$\boldsymbol{h}' = (\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\boldsymbol{D}^{-1}.$$

- The estimated factor realization is then the portfolio return

$$\widehat{f}_{t,\text{wls}} = \boldsymbol{h}'\boldsymbol{R}_t.$$

- When the portfolio $\boldsymbol{h}$ is normalized such that $\sum_i^N h_i = 1$, it is referred to as a factor mimicking portfolio.

BARRA-Type Industry Factor Model

- Consider a stylized BARRA-type industry factor model with J mutually exclusive industries.
- The factor sensitivities β_{ij} in (1) for each asset are time invariant.
- The factor betas are dummy variables indicating whether a given asset is in a particular industry.

$$\beta_{ij} = \begin{cases} 1, & \text{if asset } i \text{ is in industry } j \\ 0, & \text{otherwise.} \end{cases}$$

- The estimated value of f_{jt} will be equal to the weighted average excess return in time period t of the firms operating in industry j.

Industry Factor Model Regression

- The industry factor model with J industries is summarized as

$$\begin{aligned} R_{it} &= \beta_{i1} f_{1t} + \cdots + \beta_{iJ} f_{Jt} + \epsilon_{it} \\ \mathbb{V}(\epsilon_{it}) &= \sigma_i^2 \\ \mathbb{C}(\epsilon_{it}, f_{jt}) &= 0 \\ \mathbb{C}(f_{it}, f_{jt}) &= \sigma_{ij} \end{aligned}$$

- It is understood that there are N_j firms in the j-th industry such that

$$\sum_{j=1}^{J} N_j = N.$$

Estimation of Industry Factor Model Factors

- Consider the cross-section regression at time t:

$$\begin{aligned} \boldsymbol{R}_t &= \boldsymbol{\beta}_1 f_{1t} + \cdots + \boldsymbol{\beta}_J f_{Jt} + \boldsymbol{\epsilon}_t \\ &= \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{\epsilon}_t \\ \mathbb{E}\left(\boldsymbol{\epsilon}_t, \boldsymbol{\epsilon}_t'\right) &= \boldsymbol{D} \\ \mathbb{C}(\boldsymbol{f}_t) &= \boldsymbol{\Omega}_f \end{aligned}$$

- Since the industries are mutually exclusive it follows that

$$\boldsymbol{\beta}_j'\boldsymbol{\beta}_k = \begin{cases} N_k & \text{for } j = k \\ 0 & \text{otherwise} \end{cases}$$

Estimation of Industry Factor Model Factors (Cont'd)

- An unbiased but inefficient estimate of the factor realizations $\boldsymbol{f}_t$ can be obtained by OLS:

$$\widehat{\boldsymbol{f}}_{t,\mathrm{OLS}} = (\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{R}_t = \begin{bmatrix} \widehat{\boldsymbol{f}}_{1t,\mathrm{OLS}} \\ \vdots \\ \widehat{\boldsymbol{f}}_{Jt,\mathrm{OLS}} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1}\sum_{i=1}^{N_1} R_{it}^1 \\ \vdots \\ \vdots \\ \frac{1}{N_J}\sum_{i=1}^{N_J} R_{it}^J \end{bmatrix}$$

Estimation of Factor Realization Covariance Matrix

- Given $\widehat{\boldsymbol{f}}_{1,\mathrm{OLS}}, \ldots, \widehat{\boldsymbol{f}}_{T,\mathrm{OLS}}$, the covariance matrix of the industry factors may be computed as the time series sample covariance:

$$\overline{\boldsymbol{f}}_{\mathrm{OLS}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\boldsymbol{f}}_{t,\mathrm{OLS}}$$

$$\widehat{\boldsymbol{\Omega}}^{F}_{\mathrm{OLS}} = \frac{1}{T-1}\sum_{t=1}^{T}\big(\widehat{\boldsymbol{f}}_{t,\mathrm{OLS}} - \overline{\boldsymbol{f}}_{\mathrm{OLS}}\big)\big(\widehat{\boldsymbol{f}}_{t,\mathrm{OLS}} - \overline{\boldsymbol{f}}_{\mathrm{OLS}}\big)'$$

Estimation of Residual Variances

- The residual variances, $\mathbb{V}(\epsilon_{it}) = \sigma_i^2$, can be estimated from the time series of residuals from the T cross-section regressions.
- For each i, σ_i^2 may be estimated, for $i = 1, 2, \ldots, N$,

$$\bar{\epsilon}_{i,\mathrm{OLS}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\epsilon}_{it,\mathrm{OLS}}$$

$$\widehat{\sigma}^2_{i,\mathrm{OLS}} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\widehat{\epsilon}_{it,\mathrm{OLS}} - \bar{\epsilon}_{i,\mathrm{OLS}}\right)^2$$

Industry Factor Model: Covariance Matrix

- Let $\widehat{\boldsymbol{D}}_{\mathrm{OLS}}$ be the diagonal matrix with $\widehat{\sigma}^2_{i,\mathrm{OLS}}$ along the diagonal.
- The covariance matrix of the N assets is estimated by

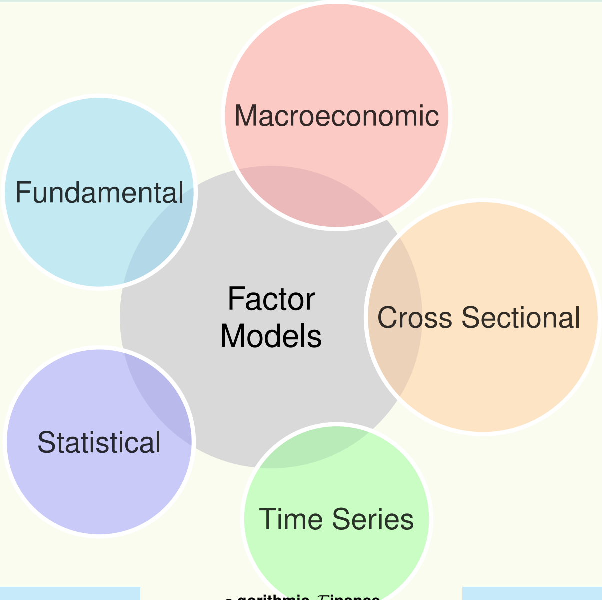
$$\widehat{\boldsymbol{\Omega}}_{\mathrm{OLS}} = \boldsymbol{B}\widehat{\boldsymbol{\Omega}}^F_{\mathrm{OLS}}\boldsymbol{B}' + \widehat{\boldsymbol{D}}_{\mathrm{OLS}}.$$

Weighted Least Squares Estimation

- The OLS estimation of the factor realizations $\boldsymbol{f}_t$ is inefficient due to the cross-sectional heteroskedasticity in the asset returns.
- The estimates of the residual variances may be used as weights for weighted least squares (**feasible GLS**) estimation:

$$
\begin{aligned}
\widehat{\boldsymbol{f}}_{t,\mathrm{GLS}} &= \left(\boldsymbol{B}'\widehat{\boldsymbol{D}}_{\mathrm{OLS}}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}'\widehat{\boldsymbol{D}}_{\mathrm{OLS}}^{-1}\boldsymbol{R}_t \\
\widehat{\boldsymbol{\Omega}}_{\mathrm{GLS}}^{F} &= \frac{1}{T-1}\sum_{t=1}^{T}\left(\widehat{\boldsymbol{f}}_{t,\mathrm{GLS}} - \overline{\boldsymbol{f}}_{\mathrm{GLS}}\right)\left(\widehat{\boldsymbol{f}}_{t,\mathrm{GLS}} - \overline{\boldsymbol{f}}_{\mathrm{GLS}}\right)' \\
\widehat{\sigma}_{i,\mathrm{GLS}}^{2} &= \frac{1}{T-1}\sum_{t=1}^{T}\left(\widehat{\epsilon}_{it,\mathrm{GLS}} - \overline{\epsilon}_{i,\mathrm{GLS}}\right)^2 \\
\widehat{\boldsymbol{\Omega}}_{\mathrm{GLS}} &= \boldsymbol{B}\widehat{\boldsymbol{\Omega}}_{\mathrm{GLS}}^{F}\boldsymbol{B}' + \widehat{\boldsymbol{D}}_{\mathrm{GLS}}.
\end{aligned}
$$

Takeaways


Macroeconomic
Fundamental
Factor Models
Cross Sectional
Statistical
Time Series

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