Information Ratio

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Broad Lesson Plan

Introduction

- **What is investment? Why investing?**
- What is **quantitative investment**?
  - select securities based on algorithmic or systematic quantitative analysis that has a higher probability of out-performance going forward
  - construct investment **portfolio** to achieve optimal performance
- Example of quantitative investment: **AI Power Equity ETF** (AIEQ)
- Another example of quantitative investment: **Vanguard Strategic Equity Fund** (VSEQX)
- BIG QUESTION: How do you measure the performance of an investment fund?

ETFs: Number of Funds by Type of Fund

		Equity						Registered		Unregistered	Fund
		Domestic									
Year	Total	Broad-Based	Sector	Global	Bond	Commodities	Hybrid	Index	Actively managed		of funds
1993	1	1	-	-	-	-	-	1	-	-	-
1994	1	1	-	-	-	-	-	1	-	-	-
1995	2	2	-	-	-	-	-	2	-	-	-
1996	19	2	-	17	-	-	-	19	-	-	-
1997	19	2	-	17	-	-	-	19	-	-	-
1998	29	3	9	17	-	-	-	29	-	-	-
1999	30	4	9	17	-	-	-	30	-	-	-
2000	80	29	26	25	-	-	-	80	-	-	-
2001	102	34	34	34	-	-	-	102	-	-	-
2002	113	34	32	39	8	-	-	113	-	-	-
2003	119	39	33	41	6	-	-	119	-	-	-
2004	152	60	42	43	6	1	-	151	-	1	-
2005	204	81	65	49	6	3	-	201	-	3	-
2006	359	133	119	85	6	16	-	343	-	16	-
2007	629	197	191	159	49	28	5	601	-	28	-
2008	728	204	186	225	62	45	6	670	13	45	15
2009	797	222	179	244	98	49	5	727	21	49	23
2010	923	243	193	298	128	55	6	843	26	54	27
2011	1,135	288	229	368	168	75	7	1,029	33	73	31
2012	1,195	275	222	404	202	79	13	1,076	42	77	44
2013	1,295	293	235	438	238	76	15	1,163	60	72	37
2014	1,412	317	236	494	264	82	19	1,232	108	72	39
2015	1,595	361	266	592	274	81	21	1,402	120	73	49
2016	1,716	396	304	629	285	80	22	1,501	149	66	58
2017	1,832	472	298	629	309	91	33	1,569	194	69	65

ETFs: Total Net Assets ($Millions) by Type of Fund

		Equity						Registered		Unregistered	Fund
		Domestic							Actively		of
Year	Total	Broad-Based	Sector	Global	Bond	Commodities	Hybrid	Index	managed		funds
1993	$464	$464	-	-	-	-	-	$464	-	-	-
1994	424	424	-	-	-	-	-	424	-	-	-
1995	1,052	1,052	-	-	-	-	-	1,052	-	-	-
1996	2,411	2,159	-	$252	-	-	-	2,411	-	-	-
1997	6,707	6,200	-	506	-	-	-	6,707	-	-	-
1998	15,568	14,058	$484	1,026	-	-	-	15,568	-	-	-
1999	33,873	29,374	2,507	1,992	-	-	-	33,873	-	-	-
2000	65,585	60,529	3,015	2,041	-	-	-	65,585	-	-	-
2001	82,993	74,752	5,224	3,016	-	-	-	82,993	-	-	-
2002	102,143	86,985	5,919	5,324	$3,915	-	-	102,143	-	-	-
2003	150,983	120,430	11,901	13,984	4,667	-	-	150,983	-	-	-
2004	227,540	163,730	20,315	33,644	8,516	$1,335	-	226,205	-	$1,335	-
2005	300,820	186,832	28,975	65,210	15,004	4,798	-	296,022	-	4,798	-
2006	422,550	232,487	43,655	111,194	20,514	14,699	-	407,850	-	14,699	-
2007	608,422	300,930	64,117	179,702	34,648	28,906	$119	579,517	-	28,906	-
2008	531,288	266,161	58,374	113,684	57,209	35,728	132	495,314	$245	35,728	$97
2009	777,128	304,044	82,053	209,315	107,018	74,528	169	701,586	1,014	74,528	824
2010	991,989	372,377	103,807	276,622	137,781	101,081	322	888,175	2,759	101,055	1,294
2011	1,048,139	400,702	108,548	245,114	184,222	109,176	377	934,232	5,039	108,868	1,575
2012	1,337,123	509,350	135,378	328,521	243,203	120,016	656	1,207,037	10,211	119,875	2,215
2013	1,674,713	761,798	202,706	398,834	245,862	64,042	1,469	1,596,580	14,267	63,866	2,561
2014	1,974,550	935,825	267,523	414,805	296,376	56,974	3,047	1,901,223	16,789	56,538	5,030
2015	2,100,658	965,338	267,355	474,640	340,270	49,317	3,738	2,029,296	22,891	48,471	10,476
2016	2,524,388	1,224,187	302,637	502,702	427,133	62,777	4,951	2,433,798	28,961	61,629	9,701
2017	3,400,696	1,603,965	374,454	792,248	553,302	68,927	7,800	3,288,485	44,924	67,288	11,944

ETFs: Net Issuance ($Millions) by Type of Fund

Year	Total	Equity			Bond	Commodities	Hybrid	Registered		Unregistered	Fund of funds
		Domestic		Global				Index	Actively managed		
		Broad-Based	Sector								
1993	$442	$442	-	-	-	-	-	$442	-	-	-
1994	-28	-28	-	-	-	-	-	-28	-	-	-
1995	443	443	-	-	-	-	-	443	-	-	-
1996	1,108	842	-	$266	-	-	-	1,108	-	-	-
1997	3,466	3,160	-	306	-	-	-	3,466	-	-	-
1998	6,195	5,158	$484	553	-	-	-	6,195	-	-	-
1999	11,929	10,221	1,596	112	-	-	-	11,929	-	-	-
2000	42,508	40,591	1,033	884	-	-	-	42,508	-	-	-
2001	31,012	26,911	2,735	1,366	-	-	-	31,012	-	-	-
2002	45,302	35,477	2,304	3,792	$3,729	-	-	45,302	-	-	-
2003	15,810	5,737	3,587	5,764	721	-	-	15,810	-	-	-
2004	56,375	29,084	6,514	15,645	3,778	$1,353	-	55,021	-	$1,353	-
2005	56,729	16,941	6,719	23,455	6,756	2,859	-	53,871	-	2,859	-
2006	73,995	21,589	9,780	28,423	5,729	8,475	-	65,520	-	8,475	-
2007	150,617	61,152	18,122	48,842	13,318	9,062	$122	141,555	-	9,062	-
2008	177,220	88,105	30,296	25,243	22,952	10,567	58	166,372	$281	10,567	$107
2009	116,469	-11,842	14,329	39,599	45,958	28,410	15	87,336	724	28,410	237
2010	117,982	28,317	10,187	41,527	29,652	8,155	144	108,136	1,716	8,129	433
2011	117,646	34,657	9,674	24,250	46,045	2,948	72	112,464	2,555	2,627	385
2012	185,399	57,744	14,307	51,896	52,318	8,889	246	171,377	4,988	9,035	505
2013	179,959	99,545	34,434	62,807	12,195	-29,870	849	205,154	4,710	-29,905	1,106
2014	240,844	102,394	40,593	46,642	51,007	-1,420	1,629	240,026	2,584	-1,766	2,365
2015	230,970	49,757	13,368	109,668	54,949	2,118	1,110	222,005	7,435	1,530	5,726
2016	283,914	147,805	19,705	20,195	83,442	11,679	1,088	266,223	6,247	11,444	-638
2017	470,800	156,459	29,533	159,771	120,933	1,603	2,500	454,235	15,437	1,128	1,058

What is Active Management?

- **Active management** involves a series of conscious decisions, based on insight, to invest or not invest in stocks within a defined universe, in order to meet a specific investment objective.
- For most active managers, the objective is to beat a **benchmark index** over a market cycle, while managing risk.

Setup

- $\check{r}_i$: Excess return (**in excess of risk-free rate**) on stock i
- R_b: Benchmark excess return
- β_i: Beta of stock i with respect to the benchmark

- Given a benchmark portfolio, the excess return on stock i can be decomposed into a systematic portion that is correlated with R_b and a residual return r_i that is not:

$$\check{r}_i = \beta_i R_b + r_i.$$

- Security residual return r_i's volatility is denoted by σ_i.

Benchmark and Actively Managed Portfolio

- The benchmark portfolio is defined by the weights, $w_{b,i}$, assigned to each of the N stocks in the investable universe.

$$R_b = \sum_{i=1}^{N} w_{b,i}\check{r}_i.$$

- The excess return on an actively managed portfolio, R_p, is determined by the weights, $w_{p,i}$, on each stock:

$$R_p = \sum_{i=1}^{N} w_{p,i}\check{r}_i.$$

Tutorial

- Show that

$$\sum_{j=1}^{N} w_{b,j}\beta_j = 1 \qquad \text{and} \qquad \sum_{j=1}^{N} w_{b,j} r_j = 0.$$

Active Return

- The managed portfolio's beta, β_p, is simply the weighted-average beta of the stocks in the managed portfolio:

$$\beta_p := \sum_{i=1}^{N} w_{p,i}\beta_i.$$

- Definition: **Active return**

$$R_a := R_p - \beta_p R_b$$

Proposition

The active return can be expressed as

$$R_a = \sum_{i=1}^{N} w_{p,i} r_i.$$

Proof

- By the definitions of active return and β_p, we obtain

$$\begin{aligned} R_a &= \sum_{i=1}^{N} w_{p,i}\check{r}_i - \sum_{i=1}^{N} w_{p,i}\beta_i R_b \\ &= \sum_{i=1}^{N} w_{p,i}\big(\beta_i R_b + r_i\big) - \sum_{i=1}^{N} w_{p,i}\beta_i R_b \\ &= \sum_{i=1}^{N} w_{p,i}\beta_i R_b + \sum_{i=1}^{N} w_{p,i} r_i - \sum_{i=1}^{N} w_{p,i}\beta_i R_b \\ &= \sum_{i=1}^{N} w_{p,i} r_i. \end{aligned}$$

Mean-Variance Trade-Off

- Consider the mean-variance utility function

$$U \equiv U(w_{p,1}, w_{p,2}, \ldots, w_{p,N}) := \mathbb{E}(R_a) - \lambda\sigma_a^2. \tag{1}$$

- $\mathbb{E}(R_a)$: (cross sectional) expected active return
- σ_a^2: variance of active return
- λ: a risk aversion parameter

- Given forecasts for the individual residual stock returns, α_i, the expected active return of the portfolio is

$$\mathbb{E}(R_a) = \sum_{i=1}^{N} w_{p,i}\alpha_i. \tag{2}$$

Mean-Variance Trade-Off (cont'd)

- Under the assumption that the residual stock returns are uncorrelated, and given $\mathbb{V}(\alpha_i) = \sigma_i^2$, the active return variance is

$$\sigma_a^2 = \sum_{i=1}^{N} w_{p,i}^2 \sigma_i^2. \tag{3}$$

- Substituting (2) and (3) into (1), we obtain

$$U = \sum_{i=1}^{N} \left(w_{p,i}\alpha_i - \lambda w_{p,i}^2 \sigma_i^2 \right).$$

- For each i, we obtain the optimal weight by the first-order condition:

$$w_{p,i}^* = \frac{\alpha_i}{\sigma_i^2} \frac{1}{2\lambda}. \tag{4}$$

Risk Aversion Parameter and Solution

- Square the optimal weights (4) and insert them in (3), we can solve for λ.

$$\sigma_a^2 = \sum_{i=1}^{N} (w_{p,i}^*)^2 \sigma_i^2 = \sum_{i=1}^{N} \left(\frac{\alpha_i}{\sigma_i^2} \frac{1}{2\lambda} \right)^2 \sigma_i^2 = \frac{1}{4\lambda^2} \sum_{i=1}^{N} \left(\frac{\alpha_i}{\sigma_i} \right)^2 ,$$

leading to

$$\frac{1}{2\lambda} = \frac{\sigma_a}{\sqrt{\sum_{i=1}^{N} \left(\frac{\alpha_i}{\sigma_i} \right)^2}} .$$

- Consequently, we can solve for the collection of optimal weights:

$$w_{p,i}^* = \frac{\alpha_i}{\sigma_i^2} \frac{\sigma_a}{\sqrt{\sum_{j=1}^{N} \left(\frac{\alpha_j}{\sigma_j} \right)^2}} . \tag{5}$$

Information Coefficient IC

- Based on Grinold's (1994) prescription, alphas are the product of **information coefficient** IC, **residual security volatility** σ_i, and **score** S_i:

$$\alpha_i = \text{IC}\, \sigma_i S_i. \tag{6}$$

- The information coefficient IC is assumed to be the same for all securities.
- Information score S_i is assumed to have zero mean and unit standard deviation.
- If follows that $\dfrac{\alpha_i}{\sigma_i} = \text{IC} S_i$ has zero mean and unit standard deviation as well.
- Consequently, $$w^*_{p,i}\sigma_i = \frac{\alpha_i}{\sigma_i}\frac{\sigma_a}{\text{IC}\sqrt{N}} = S_i \frac{\sigma_a}{\sqrt{N}}.$$

Optimal Risk Allocation

- The risk-adjusted weights $w^*_{p,i}\sigma_i$ solve the problem of optimal risk allocation.

- By construction, S_i has cross-sectional zero mean and unit standard deviation. Accordingly,

$$\text{std. dev}\left(w^*_{p,i}\sigma_i\right) = \frac{\sigma_a}{\sqrt{N}}.$$

Fundamental Law of Active Management

- Information ratio is defined as risk-adjusted alpha.

$$\text{IR} := \frac{\mathbb{E}(R_a)}{\sigma_a}.$$

- σ_a is called the **tracking error**.

Proposition: Fundamental Law of Active Management

The information ratio IR of a portfolio is equal to the size-adjusted information coefficient IC.

$$\text{IR} = \text{IC}\sqrt{N}. \tag{7}$$

Proof

- The zero-mean property of $w^*_{p,i}\sigma_i$ and α_i/σ_i allows the expected active return in (2) to be recast as follows:

$$\mathbb{E}(R_a) = \sum_{i=1}^{N} w^*_{p,i}\alpha_i = \sum_{i=1}^{N} w^*_{p,i}\sigma_i\left(\frac{\alpha_i}{\sigma_i}\right) = N\,\mathbb{C}\left(w^*_{p,i}\sigma_i, \frac{\alpha_i}{\sigma_i}\right)$$
$$= N\rho \times \mathrm{std.\,dev}(w^*_{p,i}\sigma_i) \times \mathrm{std.\,dev}\left(\frac{\alpha_i}{\sigma_i}\right) = \rho\sigma_a\sqrt{N}\,\mathrm{IC},$$

  where ρ is the correlation coefficient between risk-adjusted optimal weights and alphas, which is 1 since the two variables are proportional.

- Dividing both sides by active risk σ_a gives the original form of the fundamental law:

$$\mathrm{IR} = \mathrm{IC}\sqrt{N}.$$

Transfer Coefficient

- In practice, active managers are usually subject to constraints that cause them to deviate from the unconstrained optimal active weights, $w^*_{p,i}$.
- Let w_i be a set of constrained active weights generated by an optimizer, where the active risk is set equal to the unconstrained active risk (3).
- Definition
  TC := $\rho_{w,\alpha}$ is defined as the **transfer coefficient**.
- The transfer coefficient, TC, is calculated as the cross-sectional correlation coefficient between risk-adjusted forecast residual returns and actual weights.

Generalized Fundamental Law

Proposition

$$\text{IR} \approx \text{TC} \times \text{IC}\sqrt{N}$$

- Proof:

$$\begin{aligned}
\mathbb{E}(R_a) &= \sum_{i=1}^{N} w_i\alpha_i = \sum_{i=1}^{N}(w_i\sigma_i)\left(\frac{\alpha_i}{\sigma_i}\right) = N\,\mathbb{C}\left(w_i\sigma_i, \frac{\alpha_i}{\sigma_i}\right) \\
&= N \times \text{TC} \times \mathrm{std.\,dev}(w_i\sigma_i) \times \mathrm{std.\,dev}\left(\frac{\alpha_i}{\sigma_i}\right) \\
&= N \times \text{TC} \times \text{IC}\sqrt{\sigma_a^2/N - \overline{w_i\sigma_i}^2} \\
&\approx \sigma_a \text{TC} \times \text{IC}\sqrt{N}
\end{aligned}$$

since the cross-sectional mean $\overline{w_i\sigma_i} \approx 0$.

Illustration

Diversified Manager

- IR = 2.0
- $\sqrt{N} = \sqrt{500}$ = 22.4
- TC = 0.8

- 2.0 = 22.4$\times$ 0.8$\times$ IC
- IC = 0.11

Concentrated Manager

- IR = 2.0
- $\sqrt{N} = \sqrt{50}$ = 7.1
- TC = 0.7

- 2.0 = 7.1$\times$ 0.7$\times$ IC
- IC = 0.40

Interpretation

- The generalized “Fundamental Law” says that efficiency (return per unit of risk, as measured by IR) is a function of
  - the manager’s insight and ability to forecast returns (IC)
  - the breadth of that insight, i.e., (number (N) of forecasts generated
  - the freedom of the manager to implement those insights within the portfolio (TC)
- The stock-picking expertise required for an active manager to generate risk-efficient returns may be broad (covering many stocks) or deep (focused on only a few stocks).
- Of course, the ideal manager will have both breadth and depth which requires a seasoned and extensive research capability.

Takeaways

- ETFs are gaing more ground, even those that are called “actively managed”.
- Active managers are those who can bring the required breadth and depth of expertise to consistently and actively manage a well-diversified portfolio.
- Still, the information ratio is the main metric to assess a manager’s ability to beat the benchmark.
- IR = IC $\times \sqrt{N} \times$ TC
  Efficiency = Skill $\times$ Breadth $\times$ Implementation